Newton's Laws
Newton's Laws

• Before Isaac Newton
  • There were facts and laws about the way the physical world worked, but no explanations

• After Newton
  • There was a unified system that explained those facts and laws and many other things besides

• Newton published that system in *Mathematical Principles of Natural Philosophy* in 1686

• Among other things, the *Principia* explained motion
• And to understand the universe, you need to understand motion
• Because *everything* in the universe moves
Prelude to Newton's Laws

• How to describe motion?
  • *Position*
    • where it is
  • *Velocity*
    • how fast and in what direction it is going
  • *Acceleration*
    • how fast and in what direction its velocity is *changing*
      • when something *speeds up*, it is accelerating
      • when something *slows down*, it is accelerating
        • *(deceleration = negative acceleration)*
    • there can even be acceleration without a change of speed!…
      • don’t believe it? watch this…
Prelude to Newton's Laws

- There is a very important type of acceleration in astronomy:
  - the *acceleration due to gravity*
  - Consider the ball dropped off the building at right
  - It accelerates at a rate of about ten meters per second, per second, or $10 \text{ m/s}^2$ (more exactly, $9.8 \text{ m/s}^2$)
  - This is called the acceleration of gravity, symbolized by $g$
Prelude to Newton's Laws

• In a given gravity field, all objects experience the same gravitational acceleration
  • A piece of paper and a brass mass…
  • A brass mass and a similar-shaped wad of paper…
  • A hammer and a feather…
Prelude to Newton's Laws

The Hammer and the Feather
USAF Col David R. Scott
Apollo 15
July 26th - August 7th, 1971
Prelude to Newton's Laws

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The Hammer and the Feather
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- The hammer and feather fell because they felt a force from gravity
  - The force of gravity = weight
- There would be no weight without mass
- But is mass the same as weight?
Prelude to Newton's Laws

Mass versus Weight

- **Mass** = a measure of the amount of matter in an object
- Mass does not change no matter where the object is

- **Weight** = the result of a force exerted on the object
- The amount of weight changes with the strength of gravity
- There are two kinds of weight:
  - "Normal" weight = force of gravity
  - "Apparent" weight = force of gravity + other forces
Prelude to Newton's Laws

Normal Weight versus Apparent Weight
Prelude to Newton's Laws

Normal Weight versus Apparent Weight

- Elevator stationary or moving at constant velocity: Normal weight
- Elevator accelerating upward: Heavier-than-normal weight
Prelude to Newton's Laws

Normal Weight versus Apparent Weight
Prelude to Newton's Laws

Normal Weight versus Apparent Weight

Todo Universo by Lulu Santos
Prelude to Newton's Laws

- An object’s motion is specified by its position, velocity, and acceleration
- Newton’s Laws of Motion describe why and how things move
- Newton’s Laws are related to the concept of “momentum”
Prelude to Newton's Laws

- Galileo identified momentum as a fundamental physical property of any moving object that has mass
  - At that time, it was called “impetus”
- Momentum tends to keep an object moving with the same speed and direction
  - In other words, with the same velocity
A mathematical expression for momentum is

\[ \text{momentum} = p = \text{mass} \times \text{velocity} \]

Momentum is a vector quantity, with both magnitude and direction.

More momentum → harder to change the object’s direction and speed.

But while speed and direction can be hard to change, it can be done…

- With a push from a “non-zero net force”
- Net forces can consist of more than one force…
  ...and those forces can add to zero…
  ...but they might not…
Prelude to Newton's Laws

Momentum and Force

• If forces are applied so that they balance, then the net force is zero
• But if they don’t balance, then there is a “nonzero net force”
• And that changes momentum, which is $p = m \cdot v$...
  ...by changing velocity
• If velocity changes there must have been an acceleration

• Therefore a net force causes an acceleration...
  ...which changes momentum...
  ...and that leads us to *Newton's Laws of Motion*
Newton’s Laws of Motion

1. If the net force on an object is zero, the object’s velocity is constant.

2. A nonzero net force on an object changes the object's momentum, accelerating it in the direction of the force:

\[ F_{net} = \frac{\Delta p}{\Delta t} \quad \quad F_{net} = ma \]

3. For every force, there is an equal but opposite reaction force.

Newton's laws reflect a property of momentum called Conservation of Momentum
Conservation of Momentum

- "The total amount of momentum in the universe is constant"
- A more useful way to say it: "The total amount of momentum in an isolated system is constant"
- So how do Newton's laws of motion reflect Conservation of Momentum?
How Newton's Laws Reflect Conservation of Momentum

1. If the net force on an object is zero, the object’s velocity is constant…
   …therefore its momentum is constant (= “conserved”).

2. If there is a nonzero net force on the object...
   …the net force accelerates it in the direction of the force… \[ F_{\text{net}} = ma \]
   …changing its momentum… \[ F_{\text{net}} = \frac{\Delta p}{\Delta t} \]
   …so there’s no conservation here…
   …but the third law says…

3. “For every force there is an equal but opposite reaction force“…
   …so if object 2 exerts a force on object 1…
   …object 1’s momentum will change…
   …but object 1 will exert an equal but opposite reaction force on object 2…
   …changing object 2’s momentum by an equal but opposite amount…

→ and momentum is conserved ←
Here's a familiar example of Conservation of Momentum:

Before collision:
- First ball: momentum = \( m \times v \)
- Second ball: momentum = 0

The collision transfers momentum from the first ball to the second ball.

After collision:
- First ball: momentum = 0
- Second ball: momentum = \( m \times v \)
• Rockets are important parts of space programs
• Rockets are important parts of space programs
• What do you think makes rockets launch?
It’s Conservation of Momentum

- Before launch, the rocket body and the fuel inside together have zero momentum.
- After launch, the fuel shoots out the exhaust with large momentum.
- To conserve momentum, the rest of the rocket must move in the opposite direction.
We’ve been talking about **linear** or **translational momentum** \((p = mv)\) 

The name distinguishes it from another type of momentum… 

…one that is very important in astronomy 

Objects that are moving through space (“translating”) have **linear momentum** 

Objects that are *rotating* have this other type of momentum 

It’s called **angular momentum**
Angular Momentum

- Angular momentum is a **vector**, \( L \)
- \( L \) has **magnitude** and **direction**
- The **magnitude** is given by \( m \cdot v \cdot r \)
- The **direction**…
  
  …is perpendicular to the rotation…
  
  …and is given by the “**right hand rule**”:
  
  • Curl fingers of right hand in rotation direction
  • Thumb points in direction of \( L \)

- Angular momentum vectors sometimes appear in depictions of spinning celestial bodies
Angular Momentum
Angular Momentum

• Angular momentum is conserved
• Conservation of Linear Momentum says:
  "in the absence of a net force, the linear momentum of an isolated system remains constant"
• So to change *linear momentum*, you need a *nonzero net force*
• To change *angular momentum*, you need a *nonzero "twisting force"*…
  …also called a *"torque"*
• So Conservation of Angular Momentum can be expressed:
  "in the absence of a net torque, the angular momentum of an isolated system remains constant"
• But since we are scientific thinkers, let's test this claim…
Angular Momentum

• Here’s a good example of conservation of angular momentum *magnitude*

• A bicycle wheel and a spinnable platform can demonstrate conservation of angular momentum *direction*…

• Spinning objects like ice skaters – and bicycle wheels – are made of atoms and molecules

• They continue to spin as a unit and don’t come apart because they are held together by intermolecular forces, which are mostly electromagnetic

• True also of a ball on a string…
• If the ball spins at a constant rate it has constant angular momentum.
• The string exerts a “centripetal force” on the ball.
• The centripetal force causes the ball’s linear momentum and velocity to change constantly…
  …and it spins in a circle.
• If the ball spins at a constant rate it has constant angular momentum
• The string exerts a “centripetal force” on the ball
• The centripetal force causes the ball’s linear momentum and velocity to change constantly…
  …and it spins in a circle
• If the string breaks, the ball “flies off on a tangent”
Orbital Motion

- Planets orbit the Sun in roughly circular (actually elliptical) orbits
- Sort of like balls on strings, but there are no strings
- There still must be a centripetal force, though
- So what causes them to orbit?
- And that brings us back to…

GRAVITY
...Sir Isaac Newton and the apple....
• But it didn't happen exactly that way
• According to Newton himself, much later, he did see an apple fall
• But it didn't fall on his head and knock that equation into it
• Instead, the story goes, he noticed that even apples from the very tops of the trees fall to the ground...
• Instead, the story goes, he noticed that even apples from the very tops of the trees fall to the ground
• Then he looked up and saw the Moon, even higher than that…
• And he started thinking…
…could the same force that causes an apple to fall to the ground cause the Moon to orbit Earth?
• But that makes no sense, right?
  • An apple falls straight to the ground…
    …the Moon does not!
• But Newton wasn't thinking of things that fall *straight* down…
  …he was thinking of *projectiles*
• And projectiles *do* fall
• They’re just going *sideways*…
A cannonball dropped from rest at Earth's surface falls ~5 m in 1 s.
Earth's surface drops ~5 m in 8,000 m.
So if the cannonball travels sideways at 8,000 m/s parallel to the surface, it will never hit – it will orbit Earth.
- This speed is called *orbital velocity* or *orbital speed*
  - 8,000 m/s = 5 miles/sec = 18,000 mph)

(escape velocity = 11,000 m/s = 7 miles/sec = 25,000 mph)
Newton realized that the Moon might orbit Earth for the same reason as a cannonball with orbital velocity:
- Earth’s surface curves away at the same rate as the Moon falls
- But was it the same force that causes it to fall?
- So Newton asked if the Moon falls 5 meters in 1 second, like the cannonball
- It doesn’t…it only falls a little more than 1 millimeter in one second
  …about 1/3600th as far as the cannonball
- So the Moon is not pulled on as hard as the earthly cannonball
- (Maybe Aristotle was right after all?… …things work differently up there in the heavens?)
• Newton didn’t think so
• He agreed that the Moon was not pulled on as hard as the cannonball, but not because things work differently up there
• Instead, he proposed that the same force – gravity – attracts them both
• It’s just that the Moon isn’t attracted as much because it’s farther away
Newton was able to determine how gravity depends on distance:

- The Moon is about 60 Earth radii from Earth’s center…
- …so it’s about 60 times farther away than Earth’s surface is
- The Moon falls 1/3600th of the distance that the cannonball falls
- $3600 = 60 \times 60$
- So Newton concluded that force of gravity decreases with increasing distance as the inverse square of the distance:
  
  The Moon is 60X farther away, so it feels $1/(60 \times 60)$ of the force
• So it didn’t really happen the way the cartoon depicts
• But it seems not to have happened according to Newton’s story either
• Turns out that correspondence between Newton and Robert Hooke show that Hooke suggested the inverse square relation to Newton around 1680
• But Hooke was thinking of the motion of planets around the Sun
• Newton took it further than that, both mathematically and conceptually
• Newton said gravity worked between any two masses, as described in the Law of Universal Gravitation…
Law of Universal Gravitation

• Newton did not know the value of G
• In 1798, Henry Cavendish first measured it
• The current accepted value is

\[ G = 6.67 \times 10^{-11} \frac{m^3}{kg \cdot s^2} \]
Newton vs. Einstein

Gravity is a force caused by mass.

\[ F_g = G \frac{M_1 M_2}{d^2} \]

Gravitational forces influence the motion of matter.

Mass-energy tells spacetime how to curve.

\[ G_{\mu\nu} + \Lambda g_{\mu\nu} = \frac{8\pi G}{c^4} T_{\mu\nu} \]

Spacetime curvature tells matter how to move.
Resolution: Spacetime Curvature

- Spacetime is curved by massive bodies. Imagine a small mass rolling in on the spacetime curvature toward a massive body.

The mass of the Sun causes spacetime to curve . . .

. . . so freely moving objects (such as planets and comets) follow the straightest possible paths allowed by the curvature of spacetime.

Circles that were evenly spaced in flat spacetime become more widely spaced near the central mass.
The “why” of Kepler’s Laws

• Even without a precise value for G, Newton was able to derive Kepler’s laws from

\[ F_g = G \frac{M_1 M_2}{d^2} \]

Laws of Motion 1, 2, 3

• This explained why Kepler’s laws worked…
  …because the planets were attracted to each other by gravity!
Kepler’s First Law

- Planets move in elliptical orbits with the Sun at one focus
Kepler’s Second Law

- Planets in orbit sweep out equal areas in equal times
Kepler’s Third Law

More distant planets orbit the Sun at slower average speeds, obeying the relationship

\[ p^2 = a^3 \]

- \( p \) = orbital period in years
- \( a \) = average distance from Sun in AU
Newton’s Version of Kepler’s Third Law

\[ p^2 = a^3 \quad \text{Kepler’s version} \]

\[ p^2 = \frac{4\pi^2}{G(M_1 + M_2)} a^3 \quad \text{Newton’s version} \]

• As in Kepler’s version, \( p \) is the period and \( a \) is the average orbital distance
• But Newton’s version is more general than Kepler’s
• Kepler’s only works for the Sun and our planets
• Newton’s works for any orbiting objects
Newton’s Version of Kepler’s Third Law

\[ p^2 = \frac{4\pi^2}{G(M_1 + M_2)}a^3 \]

• In Newton’s version:
  • \( M_1 \) and \( M_2 \) are the masses of the orbiting objects (in kilograms)
  • \( G \) is the gravitational constant (in m\(^3\)/kg·s\(^2\))
  • \( p \) is in seconds, and \( a \) is in meters
• Newton’s version of Kepler’s 3\(^{rd} \) is how we get the masses of planets and stars
• Here’s how…
• Newton also found that objects orbit in ellipses with their common center of mass at one focus.

• Two objects of the same mass orbit around a common focus halfway between.
• Newton also found that objects orbit in ellipses with their common center of mass at one focus
• Two objects of the same mass orbit around a common focus halfway between
• Two objects of different mass orbit around a common focus closer to the larger mass
• Newton also found that objects orbit in ellipses with their common center of mass at one focus.
• Two objects of the same mass orbit around a common focus halfway between.
• Two objects of different mass orbit around a common focus closer to the larger mass.
• The common focus for objects of very different mass, like the Sun and a planet and similar systems, is inside the larger mass.
Types of Allowed Orbits

- Newton also found that elliptical orbits were not the only ones possible.
- An elliptical orbit is an example of a "bound" orbit.
- There are also "unbound" orbits.
- Objects on bound orbits go around and around.
- Objects on unbound orbits pass by once and never return.
- Unbound orbits have more "orbital energy" than bound orbits.
- To understand what orbital energy is, we need to learn about some types of energy.
Types of Energy

• The types of energy needed to understand orbital energy are:
  • Kinetic energy
    • Energy of motion
    • $KE = \frac{1}{2}mv^2$
  • Gravitational potential energy
    • Energy of position
    • $PE_g = mgh$
Orbital Energy

• Consider Kepler’s 2\(^{nd}\) Law:
  • Planets sweep out equal areas in equal times
• So they go faster when they are closer
• And so have more kinetic energy, \( KE = \frac{1}{2}mv^2 \), when closer
• But they have less gravitational potential energy, \( PE_g = mgh \), when they are closer
• The sum, \( KE + PE_g \), is the “orbital energy”
  • And it is conserved: orbital energy = \( KE + PE_g \) = constant
Orbital Energy

- Because orbital energy is conserved…
  orbits are stable
- Orbits *can* change…
  but only by adding or taking away energy from the object
- One way to do this is with a “gravitational encounter”
Gravitational Encounters

- Comet comes in on a high-energy unbound orbit
- Gravity of Jupiter slows it down
- Loss of energy makes it adopt a lower-energy bound orbit
Gravitational Encounters

- Gravitational encounters like this, aka "gravitational slingshots", are important in space travel:

- How are spacecraft trajectories plotted?
  - NEWTON’S LAWS
- Newton’s laws also help us understand tides…
Why do tides occur?
They are caused by the gravity of the Moon
How does that work…?
The Moon pulls harder on the nearer side
This stretches Earth out, making two tidal bulges on opposite sides
Tides, Tidal Friction, and Synchronous Rotation

- The Moon goes around Earth slower than Earth rotates.
- So any point on Earth should have two high tides and two low tides each day.
- But they aren’t exactly 12 hours apart.
- Why?
Tides, Tidal Friction, and Synchronous Rotation

• It’s because the Moon orbits around Earth.
• So at a given location, Earth has to go through more than one sidereal rotation to get back to the same tide.
• It also depends on the shape of the coast and the shape of the bottom.

Not to scale! The real tidal bulge raises the oceans by only about 2 meters.
Tides, Tidal Friction, and Synchronous Rotation

- For example, the tidal variation in mid-ocean is 2 meters (6’ 6”) or less
- At Jacksonville Beach it is about 4 feet from low to high
- Elsewhere, the variation can be much greater

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- For example, in the Bay of Fundy

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Tides, Tidal Friction, and Synchronous Rotation

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- At Jacksonville Beach it is about 4 feet from low to high.
- Elsewhere, the variation can be much greater.
- For example, in the Bay of Fundy.
- This is high tide there.
Tides, Tidal Friction, and Synchronous Rotation

- For example, the tidal variation in mid-ocean is 2 meters (6’ 6”) or less
- At Jacksonville Beach it is about 4 feet from low to high
- Elsewhere, the variation can be much greater
- For example, in the Bay of Fundy
- This is low tide
For example, the tidal variation in mid-ocean is 2 meters (6’ 6”) or less.

At Jacksonville Beach it is about 4 feet from low to high.

Elsewhere, the variation can be much greater.

For example, in the Bay of Fundy.

The tides can vary by as much as 40 feet!
Tides, Tidal Friction, and Synchronous Rotation

- This is due to the shape of the bay
- When in a confined space like a bay – or a bathtub – water wants to slosh back and forth with a particular frequency
- In the Bay of Fundy, the tides roll in and out at the same frequency the water wants to slosh
This is due to the shape of the bay. When in a confined space like a bay – or a bathtub – water wants to slosh back and forth with a particular frequency. In the Bay of Fundy, the tides roll in and out at the same frequency the water wants to slosh. So the sloshing amplifies the tides and leads to the huge variation in water height between low and high tides.
Tides, Tidal Friction, and Synchronous Rotation

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So the sloshing amplifies the tides and leads to the huge variation in water height between low and high tides
Hopewell Rocks
New Brunswick, Canada
Hopewell Rocks
New Brunswick, Canada
45.6-foot tide
Tides, Tidal Friction, and Synchronous Rotation

- The Sun also affects the tides, but because of its distance, only about 1/3 as much as the Moon.
- Occasionally the Sun and the Moon work together to produce unusually extreme tides.
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When the Sun, Earth, and Moon are in a line there is a “spring tide.”
The Sun also affects the tides, but because of its distance, only about 1/3 as much as the Moon.

Occasionally the Sun and the Moon work together to produce unusually extreme tides.

When the Sun, Earth, and Moon are in a line there is a “spring tide.”

When they form a right angle there is a “neap tide.”
But in fact, the tidal bulge is not lined up with Earth and Moon. This is due to friction between the solid Earth and the water above.
Tides, Tidal Friction, and Synchronous Rotation

- Earth’s rotation pulls the tidal bulges along
- This causes them to run slightly “ahead” of the Earth-Moon line
- If Earth didn’t rotate faster than the Moon orbits, the bulges would be right on the Earth-Moon line
Tides, Tidal Friction, and Synchronous Rotation

- The Moon’s gravity pulls back on the bulge, slowing Earth’s rotation
  - As a result, the length of a day increases ~2 ms/century (1 s/50,000 y)
What about the effect of the Moon on Earth?

- If the Moon pulls on Earth’s tidal bulge, what does Newton’s 3rd Law say?
- It says the bulge exerts an equal but opposite force on the Moon.
- This pulls the Moon ahead in its orbit, increasing its orbital energy.
- As a result, the Moon moves away from Earth by ~4 cm per year.
- So the Moon is almost 2 m farther away than when Apollo 11 landed.
How would these Earth-Moon interactions affect their angular momentum?

- If Earth’s rotation slows, it loses angular momentum
- But the angular momentum of the system is conserved
- So the angular momentum lost by Earth is gained by the Moon
- Which is why its orbital energy increases, and it moves away
• Earth’s rotation is slowed only slightly by this process
• But Earth’s tidal force causes a tidal bulge on the Moon, which Earth pulls on
• The Moon is much less massive, so its rotation has been affected much more
So over time, the Moon’s rotation has slowed so much that it matches its orbital period. It is now in “synchronous rotation” with its orbit. This is why it always shows the same face to us.
Tides, Tidal Friction, and Synchronous Rotation

• Well, almost the same face
• The wobbling is called “libration”, and it’s caused by three things:
  • The eccentricity of the Moon’s orbit and Kepler’s 2nd Law (“longitudinal”)
  • The tilt of the Moon’s orbit (“latitudinal”)
  • Our movement from side to side as Earth rotates (“diurnal”)
The Moon’s synchronous rotation is an example of a 1:1 orbital:rotational resonance…
…it orbits Earth in the same time it takes to spin once around its rotation axis.
Such orbital resonances are caused by tidal forces and tidal friction…
…and they are common…
• For example, Pluto and Charon are in synchronous rotation with each other
• They both spin once and orbit their common center of mass once in the same time
• Earth and the Moon will eventually be this way, too…
  …in about 50 billion years…
  …long after the Sun becomes a red giant potentially consuming Earth…
  …and then ends its life as a white dwarf
1:1 orbital:rotational resonances are the simplest type
More complicated types exist, though…
…for example, Mercury’s solar day can be two Mercury years long:
http://sciencenetlinks.com/interactives/messenger/or/OrbitRotation.html