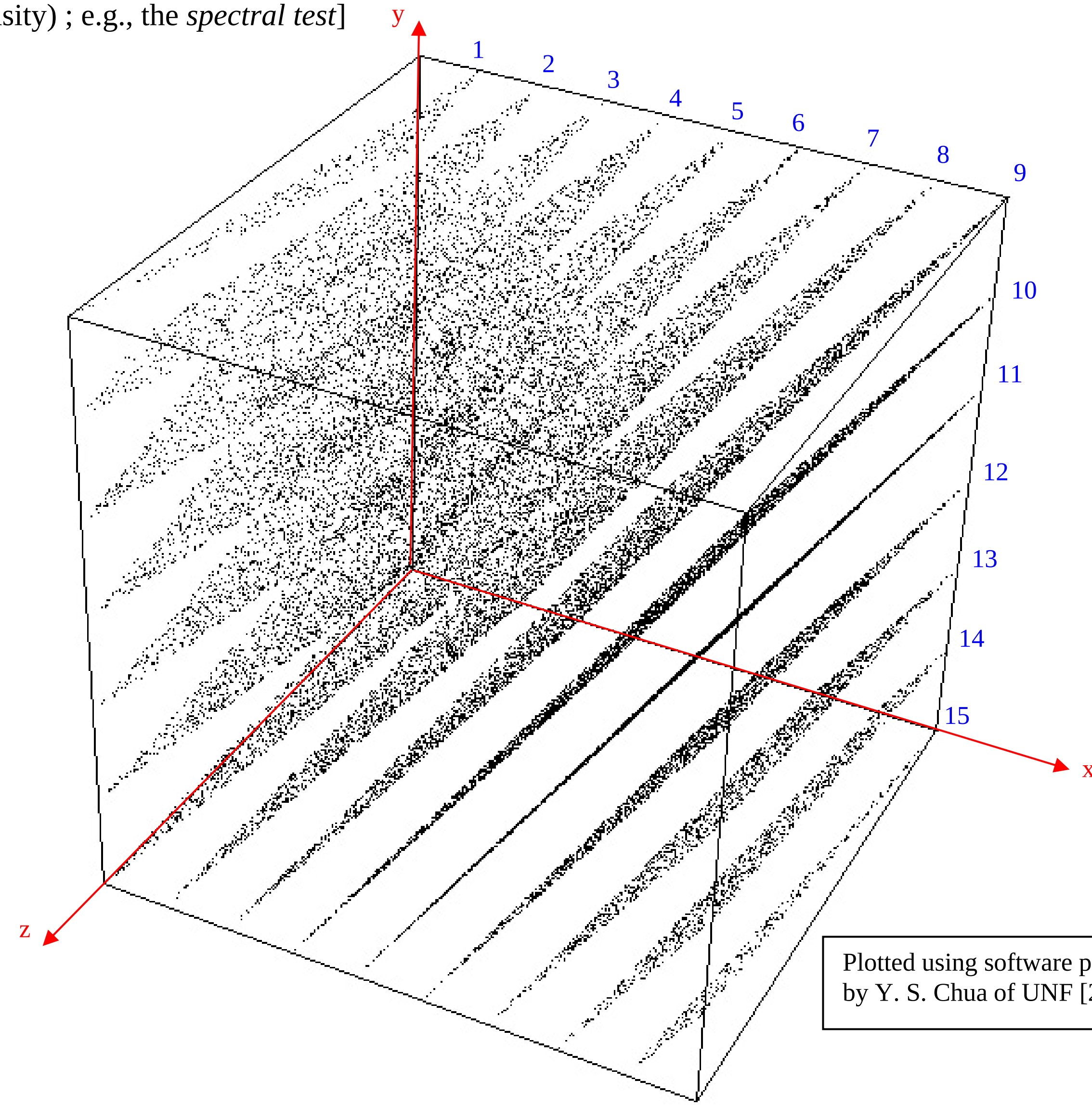
# RANDU: A Notorious RNG
## (IBM, SYSTEM/360)

*[It's still out there! Ref: Compaq Fortran Language Reference Manual (Order Number: AA-Q66SD-TK) September 1999 (formerly DIGITAL Fortran and DEC Fortran 90)]*

- RANDU, an LCG with $m = 2^{31}$, $a = 2^{16} + 3 = 65539$, $c = 0$ has a major failing which became apparent as per Marsaglia's (1968) observation regarding LCG random numbers that "*random numbers fall mainly in the planes*" [*Proceedings of the National Academy of Sciences*, 61, pp. 25-28 (1968)].
- Marsaglia's main result states that n-tuples produced using successive values drawn from an LCG lie in at most $(n!m)^{1/n}$ parallel (n -1)-dimensional hyperplanes in n-space. For n=3, this implies that with $m=2^{31}$ the points will fall in at most 2,344 parallel (2-dimensional) planes.
- The values chosen for RANDU are particularly unfortunate due to the fact that for a triple of successive values $(x_i, x_{i+1}, x_{i+2})$, $7x_i - 6x_{i+1} + x_{i+2} = 0 \pmod{2^{31}}$

  $[9x_i - 6x_{i+1} + x_{i+2} = 9x_i - 6ax_i + a^2x_i = (a-3)^2\, x_i = (2^{16})^2\, x_i = 2^{31}\, 2x_i = 2x_i \pmod{2^{31}})]$

  ie., each triple lies on one of a set of parallel planes, $2^{31}$ apart (15 of which intersect the $2^{31}\times2^{31}\times2^{31}$ cube containing the generated triples).
- A plot of 32,000 triples of the form $(x_0, x_1, x_2), (x_3, x_4, x_5), \ldots$ taken from 96,000 consecutive RANDU values demonstrate that they "clump" along a mere 15 parallel planes! [Marsaglia's paper led to the development of tests for checking for *n-space uniformity* (or density) ; e.g., the *spectral test*]




Plotted using software provided by Y. S. Chua of UNF [2004]