3 Raytrace: In taking a picture of a scene, you need to define the following parameters:

- \( c \) - the camera center, a point written in world coordinates.
- \( w \) - the direction of the optical axis, a unit vector that points into the camera from the scene.
- \( \beta_u \) and \( \beta_v \) - the scaling of the viewed rectangle: specifically, if you are viewed rectangle \( k \) units away from the camera should have dimensions \((\beta_u k) \times (\beta_v k)\).
- \( m \) and \( n \) - the dimensions of the image.
- the orientation of the horizontal and vertical axes of the image.

We will also make the following assumptions:

- The ratios \( \beta_u / \beta_v \) and \( m/n \) are equal.
- The optical axis is not parallel to the \( y \)-axis. The horizontal axis of the image will be matched up with \( j \times w \), where \( j = \langle 0, 1, 0 \rangle \). The vertical axis of the image will be matched up with \( w \times (j \times w) \).

Write \( c = (c_1, c_2, c_3) \) and \( w = \langle \cos \theta \sin \phi, \cos \phi, \sin \theta \sin \phi \rangle \). Determine the projection matrix that satisfies all of the above. Your formula will be in terms of \( c_1, c_2, c_3, \theta, \phi, \beta_u, \beta_v, m, \) and \( n \).

(NOTE 1: The second assumption mentions \( j \) - let’s explain why. The positive \( y \)-axis is “up” for us. When you hold a camera, you typically align the camera so that the camera’s “up” is as close to the world’s “up” as possible. The second assumption is the mathematical way of saying this. If the optical axis was parallel to the \( y \)-axis, then you would be taking a picture while looking straight up or while looking straight down.)

(NOTE 2: The form for \( w \) is the general form for 3-D unit vector. I could have had \( w = \langle w_1, w_2, w_3 \rangle \), but then you’d have to worry about the length of \( w \).)

(NOTE 3: The first assumption says that your picture will not be distorted. If the ratios were not the same, then your picture would get stretched or squished (Mathematica does this all the time when you do a plot, unless you reset the AspectRatio).