16.4. Just after the fan is turned on, the motor gives the blade an angular acceleration \( \alpha = 0.06t \) rad/s\(^2\), where \( t \) is in seconds. Determine the speed of the tip \( P \) of one of the blades when \( t = 3 \) s. How many revolutions has the blade turned in 3 s? When \( t = 0 \) the blade is at rest.

\[
\alpha = \frac{d\omega}{dt} = 0.06t \text{ rad/s}^2
\]

\[
\omega = \int 0.06t \, dt = 33.3(1 - e^{-0.06t}) \text{ rad/s}
\]

\[
\omega = 17.71 \text{ rad/s}
\]

\[
\phi = \int 0.3(1 - e^{-0.06t}) \, dt = 1.96 \text{ rad}
\]

\[
\phi = 33.3 \text{ rad} = 5.94 \text{ rev}
\]

16.5. Due to an increase in power, the motor \( \mathcal{M} \) rotates the shaft \( A \) with an angular acceleration of \( \alpha = \frac{0.005t}{t + 1} \) rad/s\(^2\), where \( t \) is in radians. If the shaft is initially turning at \( \omega = 50 \) rad/s, determine the angular velocity of point \( B \) after the shaft undergoes an angular displacement \( \Delta \phi = 10 \) rev.

\[
\omega = \alpha \Delta \phi = \frac{0.005t}{t + 1} \Delta \phi
\]

\[
\int \frac{0.005t}{t + 1} \, dt = 0.35 \Delta \phi
\]

\[
0.5^2 \approx 0.75 \text{ rad/s}
\]

\[
\omega = 111.45 \text{ rad/s}
\]

\[
\Delta \phi = 10 \text{ rev}
\]

\[
\omega = 22.3 \text{ m/s}
\]
16-17. For the outboard motor in Prob. 16-16, determine the magnitudes of the velocity and acceleration at a point P located on the tip of the propeller at the instant \( t = 0.75 \) s.

\[
\begin{align*}
\omega_P &= 285.8 \text{ rad/s}^2 \\
\omega_P^2 &= 19281 \text{ rad}^2/s^2 \\
\omega &= \sqrt{285.8^2 + 19281} \\
&= 2925.6 \times 10^3 \text{ rad}^2/s^2 \\
\omega &= 150 \sqrt{2} \text{ rad/s} \\
\alpha_P &= 2 \omega_P \omega \\
&= 2 \times 19281 \times 150 \sqrt{2} \\
\alpha &= 143 \text{ in/s}^2 \\
\alpha &= 150 \sqrt{2} \text{ in/s}^2
\end{align*}
\]

16-18. Starting from rest when \( t = 0 \), pulley A is given an angular acceleration \( \omega = (60) \text{ rad/s}^2 \), where \( \theta \) is in radians. Determine the speed of block B when it has risen \( x = 6 \) m. The pulley has an inner hub D which is fixed to C and turns with it.

\[
\begin{align*}
\theta &= 60 \text{ rad} \\
\theta &= 60 \times 0.15 \text{ rad} \\
\theta &= 240 \text{ rad} \\
\theta &= 60 \times 0.15 \\
\theta &= 240 \text{ rad} \\
\theta &= 60 \times 0.15 \\
\theta &= 240 \text{ rad} \\
\alpha &= 60 \text{ rad/s}^2 \\
\alpha &= 60 \times 0.15 \text{ rad/s}^2 \\
\alpha &= 90 \text{ rad/s}^2 \\
\alpha &= 60 \text{ rad/s}^2 \\
\alpha &= 90 \text{ rad/s}^2 \\
\alpha &= 20 \text{ rad/s}^2
\end{align*}
\]
16-24. The disk starts from rest and is given an angular acceleration \( \alpha = (100 \text{ rad/s}^2) \) rad/s\(^2\), where \( \theta \) is in radians. Determine the angular velocity of the disk and its angular displacement when \( t = 4 \) s.

\[
\begin{align*}
\alpha &= 100 \text{ rad/s}^2 \\
\omega &= 0 \\
\int_{0}^{\omega} \alpha \, d\theta &= \int_{0}^{\theta} 100 \, d\theta \\
\frac{1}{2} \theta &= 100 \theta^2 \\
\theta &= \frac{\sqrt{4}}{2} = 2 \sqrt{2} \\
\theta &= 2.83 \text{ rad} \\
\omega &= 2 \sqrt{2} \text{ rad/s} \\
\omega &= 4 \text{ rad/s} \\
\theta &= 2.83 \text{ rad} \\
\end{align*}
\]

Ans

16-25. The disk starts from rest and is given an angular acceleration \( \alpha = (100 \text{ rad/s}^2) \) rad/s\(^2\), where \( \theta \) is in radians. Determine the magnitudes of the normal and tangential components of acceleration of a point \( P \) on the rim of the disk when \( t = 4 \) s.

\[
\begin{align*}
\alpha &= 100 \text{ rad/s}^2 \\
\omega &= 0 \\
\int_{0}^{\omega} \alpha \, d\theta &= \int_{0}^{\theta} 100 \, d\theta \\
\frac{1}{2} \theta &= 100 \theta^2 \\
\theta &= \frac{\sqrt{4}}{2} = 2 \sqrt{2} \\
\theta &= 2.83 \text{ rad} \\
\omega &= 2 \sqrt{2} \text{ rad/s} \\
\omega &= 4 \text{ rad/s} \\
\end{align*}
\]

Ans
16-20. Rotation of the robotic arm occurs due to linear movement of the hydraulic cylinders A and B. If cylinder A is extending at the constant rate 0.5 ft/s while B is held fixed, determine the magnitude of velocity and acceleration of the part C held in the grip of the arm. The gear at D has a radius of 0.10 ft.

Angular Motion: The angular velocity of gear D must be determined first. Applying Eq. 16-8, we have

\[ \omega_D = \omega_A \cdot \frac{r_A}{r_D} = 0.5 \times \frac{0.10}{6} = 0.00833 \text{ rad/s} \]

Motion of Part C: Since the shaft that turns the robot arm is attached to gear D, then the angular velocity of the robot's arm \( \theta_D = \omega_D = 0.00833 \text{ rad/s} \). The distance of part C from the rotating shaft is \( r_C = 4.5^\circ \times 2\pi \times \frac{4.5}{360} = 0.0477 \text{ ft} \). The magnitude of the velocity of part C can be determined using Eq. 16-8

\[ v_C = r_C \cdot \omega_D = 0.0477 \times 0.00833 = 0.00397 \text{ ft/s} \]

The tangential and normal components of the acceleration of part C can be determined using Eqs 16-11 and 16-12 respectively.

\[ a_T = \omega_D \cdot r_C = 0.00833 \times 0.0477 = 0.00040 \text{ ft/s}^2 \]

\[ a_N = \frac{v_C}{r_C} = \frac{0.00397}{0.0477} = 0.0834 \text{ ft/s}^2 \]

At the instant shown, gear A is rotating with a constant angular velocity \( \omega_A = 6 \text{ rad/s} \). Determine the largest angular velocity of gear B and the maximum speed of point C.

\[ \theta_B = 10^\circ \times \frac{2\pi}{360} = 0.00556 \text{ rad} \]

\[ \omega_B = \frac{\omega_A}{\theta_B} = \frac{6}{0.00556} = 1084.3 \text{ rad/s} \]

\[ v = v_B \omega_B = 6 \times 1084.3 = 650.6 \text{ ft/s} \]

\[ v = 650.6 \text{ ft/s} \]
16-33. The bar $BC$ rotates uniformly about the shaft as $D$ with a constant angular velocity $\omega$. Determine the velocity and acceleration of the bar $AD$, which is confined by the guides to move vertically.

\[ y = 3b \cos \theta \]
\[ y' = 3b \omega \sin \theta \]
\[ y'' = 3b \omega^2 \cos \theta \]
\[ \sin \theta = \frac{v_c}{3b \omega} \text{ and } \theta = 0 \Rightarrow \theta = 90^\circ \text{ Ans} \]
\[ v_c = 3b \omega \cos 90^\circ = 0 \text{ m/s} \]
\[ a_c = 3b \omega^2 \cos 90^\circ = 0 \text{ m/s}^2 \text{ Ans} \]

16-34. The scaffold $XY$ is raised hydraulically by moving the roller at $A$ toward the pin at $B$. If $A$ is approaching $B$ with a speed of 1.5 ft/s, determine the speed at which the platform is rising at a fraction of $D$. The 4 ft links are pin-connected at their midpoints.

Potential energy relation:

\[ z = 8 \cos \theta \]
\[ y = 4 \cos \theta \]

Force equilibrium:

\[ z = 8 \cos \theta \]
\[ y = 4 \cos \theta \]
\[ z = 8 \cos \theta \]
\[ 2\theta = 1.5 \text{ ft/s} \]
\[ \theta = 0.524 \text{ radians} \]
\[ z = 8 \cos 0.524 = 6.375 \text{ ft} \]
\[ y = 4 \cos 0.524 = 3.805 \text{ ft} \]
\[ (L/2)^2 = \frac{1}{2} z \theta^2 \]
\[ \theta = 0.524 \text{ radians} \text{ Ans} \]

16-35. The mechanism is used to convert the constant circular motion $\omega$ of rod $AB$ into oscillating motion of rod $CD$. Determine the velocity and acceleration of $CD$ for any angle $\theta$ of $AB$.

\[ z = 8 \cos \theta \]
\[ z' = -8 \omega \sin \theta \]
\[ z'' = -8 \omega^2 \cos \theta \]

Using $z = 8 \cos \theta$, $\theta = \omega t$, and $t = 0$, $\theta = 0$.
\[ z'' = -8 \omega^2 \cos \theta = -8 \omega^2 \cos 0 \text{ Ans} \]
\[ z'' = -8 \omega^2 \cos \theta = -8 \omega^2 \cos \theta \text{ Ans} \]

Negative signs indicate the direction, $z''$, and $\omega$ are defined opposite in sign.
16-39. At the instant $\theta = 50^\circ$, the slotted guide is moving upward with an acceleration of 3 m/s² and a velocity of 2 m/s. Determine the angular acceleration and angular velocity of link $AB$ at this instant. Note: The upward motion of the guide is in the negative y direction.

$$y = 0.35 m$$

$$v_y = -0.23 m/s$$

$$a_v = a \left( \cos 2\theta = \cos 100^\circ \right)$$

Here, $v_y = 3 m/s$, $a_v = 3 m/s^2$, and $a = \alpha = \omega = \omega = 50^\circ$.

$$\omega = \sqrt{\frac{v^2}{r}} = \sqrt{\frac{(3 m/s)^2}{2 m}} \quad \text{Ans}$$

$$\alpha = \frac{a}{r \cos 2\theta} = \frac{3 m/s^2}{2 m \cos 100^\circ} \quad \text{Ans}$$

**16-40.** Disk $A$ rolls without slipping over the surface of the fixed cylinder $B$. Determine the angular velocity of $A$ if its center $C$ has a speed $v_C = 5 m/s$. How many revolutions will $A$ have made about its center just after link $DC$ completes one revolution?

As shown by the construction, as $A$ rolls through the arc $x = r_1$.

the center of the disk moves through the same distance $y = r_2$.

Here:

$$x = r_1$$

$$y = r_2$$

$$5 = 5 \times (0.15)$$

$$5 = 3.33 m/s$$  \text{Ans}

Last:

$$\omega = \frac{v}{r} = \frac{5 m/s}{2 m} = \omega_2$$

$$\Delta \theta = 2 \pi$$

This, $A$ makes 2 revolutions for each revolution of $CD$.  \text{Ans}

16-41. Arm $AB$ has an angular velocity of $\omega$ and an angular acceleration of $\alpha$. If no slipping occurs between the disk and the fixed curved surface, determine the angular velocity and angular acceleration of the disk.

$$\omega = \frac{d \theta}{dt} = \frac{\omega_0}{2}$$

$$\alpha = \frac{d \omega}{dt} = \frac{\alpha}{2}$$  \text{Ans}

$$\frac{d \omega}{dt} = \frac{\omega_0}{2}$$  \text{Ans}
16-44. The pins at \( A \) and \( D \) are confined to move in the vertical and horizontal tracks. If the slotted arm is causing \( A \) to move downward at \( v_A \), determine the velocity of \( B \) at the instant shown.

**Forces: continuous equation:**

\[ m \ddot{B} = \frac{d}{2} \]

**Time derivatives:**

\[ \dot{r}_1 = \left( \frac{1}{2} \right) \dot{v}_1 \]

\[ \dot{r}_2 = \left( \frac{1}{2} \right) \dot{v}_2 \]

\[ v_A = \left( \frac{1}{2} \right) v_A \]

16-45. Bar \( AB \) rotates uniformly about the fixed pin \( A \) with a constant angular velocity \( \omega \). Determine the velocity and acceleration of block \( C \) at the instant \( \phi = 60^\circ \).

\[ L \cos \phi + L \cos \phi - L \]

\[ \sin \phi + \sin \phi = 0 \]

\[ \dot{x} + \dot{x} = 0 \]

\[ \sin (2\phi) + \sin (2\phi) = \cos (\phi) + \cos (\phi) = 0 \]

When \( \phi = 60^\circ \), \( \dot{x} = 0 \).

\[ \dot{x} = -1.13L \]

\[ \dot{x} = 0 \]

**Ans:** \( v = L \cos(60^\circ) \)

**Ans:** \( \ddot{x} = \frac{a}{2} \)

\[ \dot{x} = \frac{a}{2} \]

\[ \ddot{x} = 0 \]

\[ v_x = L \cos(60^\circ) - a = \cos(60^\circ) \]

**Ans:** \( \frac{a}{2} \)

\[ a_x = -1 \cdot \sin(60^\circ) \]

\[ a_y = \cos(60^\circ) \cos(60^\circ) \]

**Ans:** \( a = 0.57L \)
16-54. The shaper mechanism is designed to give a slow cutting stroke and a quick return to a blade attached to the slider at C. Determine the velocity of the slider block C at the instant \( \theta = 60^\circ \), if link AB is rotating at 6 rad/s.

16-55. Determine the velocity of the slider block at C at the instant \( \theta = 45^\circ \), if link AB is rotating at 4 rad/s.

\[ v_C = v_B = 6 \text{ m/s} \]

\[ v_C = -0.60 \text{ m/s} \]

\[ v_B = -0.60 \text{ m/s} \]

**16-56.** The velocity of the slider block C is 4 ft/s up the inclined groove. Determine the angular velocity of links AB and BC and the velocity of point B at the instant shown.

For link BC

\[ v_B = 4 \text{ ft/s} \]

\[ v_B = v_C - v_{BC} \]

\[ v_{BC} = 4 \text{ ft/s} \]

\[ \theta = (45 + 45) \]

\[ v_B = v_C - v_{BC} \]

\[ -4\cos(45^\circ) \text{ m/s} \]

\[ -4\cos(45^\circ) \text{ m/s} \]

\[ -4\cos(45^\circ) \text{ m/s} \]

\[ -4\cos(45^\circ) \text{ m/s} \]

\[ v_B = 2.83 \text{ m/s} \]

\[ \theta = 90^\circ \text{ rad/s} \]

\[ v_B = 2.83 \text{ m/s} \]

For link AB: Link AB rotates about the fixed point, hence

\[ v_B = \omega \times r_B \]

\[ v_B = 2.83 \text{ m/s} \]

\[ \omega = 2.83 \text{ rad/s} \]
The rotation of link AB creates an oscillating movement of gear F. If AB has an angular velocity of \( \omega_{AB} = 6 \text{ rad/s} \), determine the angular velocity of gear F at the instant shown. Gear E is rigidly attached to arm CD and pivoted at D to a fixed point.

**Kinematic Diagram:** Since link AB and arm CD are rotating about the fixed points A and D respectively, then \( v_A \) and \( v_D \) are always directed perpendicular to the respective arms with the magnitudes of \( v_A = \omega_{AB} \times r_A = 6 \times 0.075 \) in/s and \( v_D = \omega_{CD} \times r_D = 1.5 \times 0.09 \) in/s. At the instant shown, \( v_A \) and \( v_D \) are directed toward negative x-axis.

**Velocity Equations:** Here, \( v_F = (0 \text{ in/s}) \pm (0.1 \text{ in/s}) \) m. Applying Eq. 18 to 19, we have:

\[
\begin{align*}
\vec{v}_F &= \vec{v}_A + \vec{v}_D + \vec{a}_F \\
-0.435 &= -0.156x_F + 0.156x_F + 0.000001 + 0.0013 \\
-0.435 &= 0.0013 \\
0.0013 &= 0.0013 \\
\end{align*}
\]

Equating i and j components gives:

\[
\begin{align*}
0 &= 0.000001 \quad \omega_{CF} = 0 \\
-0.435 &= 0.0013 \quad \omega_{DF} = 3.00 \text{ rad/s}
\end{align*}
\]

**Angular Motion About a Fixed Point:** The angular velocity of gear E is determined with arm CD since they are attached together. Then, \( \omega_D = \omega_{CD} + 3.00 \text{ rad/s} \). Here, \( \omega_{DF} = \omega_{DF} \) where \( \omega_{DF} \) is the angular velocity of gear F.

\[
\omega_F = \frac{2\omega_D}{25} \times 0.030 = 12.5 \text{ rad/s}
\]

Ann
16-61. At the instant shown, the truck is traveling to the right at \( v_r = 3 \text{ m/s} \), while the pipe is rolling counterclockwise at \( \omega_p = 8 \text{ rad/s} \) without slipping at \( B \). Determine the velocity of the pipe's center \( G \).

\[
v_r = v_x + v_{rG}
\]

\[
v_{rG} = \frac{\gamma}{2} + \omega_p
\]

\[
v_r = 3 \text{ m/s} == \text{ Ans}
\]

Also:

\[
v_x = v_x + \gamma \times v_{rG}
\]

\[
v_{rG} = \frac{\gamma}{2} + (8 \text{ rad/s}) \times (1.5 \text{ m})
\]

\[
v_G = 3 - 12
\]

\[
v_G = -9 \text{ m/s} == \text{ Ans}
\]

16-62. At the instant shown, the truck is traveling to the right at \( v_r = 8 \text{ m/s} \). If the spool does not slip at \( B \), determine its angular velocity so that its mass center \( G \) appears to an observer on the ground to remain stationary.

\[
v_G = v_x + v_{rG}
\]

\[
0 = \gamma + 1.5 \omega_p
\]

\[
\omega_p = \frac{1}{1.5} = 5.33 \text{ rad/s} \quad \text{Ans}
\]

Also:

\[
v_x = v_x = \omega_p \times v_{rG}
\]

\[
0 = \frac{8 \text{ m/s}}{1.5} \times (8 \text{ rad/s}) \times (1.5 \text{ m})
\]

\[
\omega_p = \frac{8}{1.5} = 5.33 \text{ rad/s} \quad \text{Ans}
\]
Mechanical toy animals often use a walking mechanism as shown idealized in the figure. If the driving crank AB is propelled by a spring motor such that $\omega_{AB} = 5 \text{ rad/s}$, determine the velocity of the rear foot E at the instant shown. Although not part of this problem, the upper end of the foreleg has a slotted guide which is constrained by the fixed pin at G.

\[ v_E = v_{x} + v_{oy} \]
\[ v_{x} = 2.5 + 3w \]
\[ v_{oy} = 2.21 \text{ in/s} \]
\[ \omega_{AB} = 5 \text{ rad/s} \]
\[ v_{E} = (2.21)(1.5) = 3.32 \text{ in/s} \]

Also:
\[ v_{x} = v_{dx} + v_{xy} \]
\[ v_{dx} = 4.5 \text{ in/s} \]
\[ v_{xy} = 0 \text{ in/s} \]
\[ v_{E} = v_{x} + \theta \times v_{oy} \]
\[ (\theta_{AB}) = \omega_{AB} \times \theta_{AB} \]
\[ (\theta_{AB}) = 5 \text{ rad/s} \times \theta_{AB} \]
\[ \omega_{AB} = 5 \text{ rad/s} \]
\[ v_{E} = 22.31 \text{ in/s} \]

Assume...
16-88. The wheel rolls on its hub without slipping on the horizontal surface. If the velocity of the center of the wheel is \( v_C = 2 \text{ ft/s} \) to the right, determine the velocities of points \( A \) and \( B \) at the instant shown.

\[
\begin{align*}
\omega &= \sqrt{\frac{v_C}{R}} \\
\alpha &= \frac{v_C}{R} = \left( \frac{2}{12} \right) = 0.17 \text{ rad/s} \\
\gamma &= \alpha R = \left( \frac{11}{12} \right) = 0.73 \text{ ft/s} \\
v_A &= \sqrt{\frac{v_C}{R}} = \sqrt{2} = 2.83 \text{ ft/s} \\
\theta &= \tan^{-1} \left( \frac{\frac{1}{2}}{12} \right) = 43^\circ 
\end{align*}
\]

16-89. If link \( CD \) has an angular velocity of \( \omega_{CD} = 6 \text{ rad/s} \) and the angular velocity of link \( AB \) at instant shown.

\[
\begin{align*}
v_C &= \omega_{CD} r_{CD} = (6)(0.5) = 3.0 \text{ m/s} \\
\gamma &= \alpha d_{CD} = 0.6 \text{ rad/s} \\
v_C &= \frac{1}{\cos 30^\circ} = 1.04 \left( \frac{6}{\cos 30^\circ} \right) = 7.20 \text{ m/s} \\
v_D &= \frac{1}{\cos 60^\circ} = 2 \left( \frac{3}{\cos 60^\circ} \right) = 6 \text{ rad/s} \\
v_C &= \frac{1}{\cos 30^\circ} = 1.04 \left( \frac{6}{\cos 30^\circ} \right) = 7.20 \text{ m/s} \\
v_D &= \frac{0.1}{6 \text{ rad/s}} = 40 \text{ ft/s} 
\end{align*}
\]
16-97. Due to slipping points $A$ and $B$ on the rim of the disk have the velocities shown. Determine the velocities of the center point $C$ and point $E$ at this instant.

![Diagram showing a disk with points $A$, $B$, $C$, and $E$ with velocities at $A$ and $B$.]

\[ v_A = 10 \text{ cm/s} \]
\[ v_B = 5 \text{ cm/s} \]
\[ x = 1.06647 \text{ m} \]
\[ \theta = \frac{12}{10} \text{ rad} \]
\[ v_C = \theta \cdot v_B \]
\[ v_C = 0.3751 \text{ rad/s} \cdot 5 \text{ cm/s} \]
\[ v_C = 1.8755 \text{ cm/s} \]
\[ v_E = \theta \cdot v_A \]
\[ v_E = 0.3751 \text{ rad/s} \cdot 10 \text{ cm/s} \]
\[ v_E = 3.751 \text{ cm/s} \]

16-98. The mechanism used in a marine engine consists of a single crank $AB$ and two connecting rods $BC$ and $BD$. Determine the velocity of the piston at $C$ the instant the crank is at the position shown and has an angular velocity of $5$ rad/s.

![Diagram showing a mechanism with crank $AB$, connecting rods $BC$ and $BD$, and a piston at $C$.]

\[ v_p = 3.25 \text{ ft/s} \]

![Diagram showing an analysis for point $C$.]

\[ v_{OC} = 0.4 \text{ m/s} \]
\[ r_{OC} = 0.4 \text{ m} \]
\[ v_{BC} = 0.4 \text{ m/s} \]
\[ r_{BC} = 0.4 \text{ m} \]
\[ v_{DC} = 0.3444 \text{ m/s} \]
\[ \omega = \frac{1}{10} \text{ rad/s} \]
\[ v_C = 0.4099 \text{ m/s} = 0.69 \text{ ft/s} \]
16-102. The epicyclic gear train is driven by the rotating link DE, which has an angular velocity \( \omega_D = 5 \text{ rad/s} \). If the ring gear \( F \) is fixed, determine the angular velocities of gears \( A, B, \) and \( C \).

\[ \begin{align*}
\omega_A &= 0.14(3) = 0.42 \text{ rad/s} \\
\omega_B &= 0.68 \text{ rad/s} \\
\omega_C &= 0.48 \text{ rad/s} \\
\omega_D &= 0.68 \text{ rad/s} \\
\omega_E &= 20.73 \text{ rad/s} \\
\omega_F &= 14.8 \text{ rad/s}
\end{align*} \]

16-103. The mechanism produces intermittent motion of link \( AB \). If the sprocket \( S \) is turning with an angular velocity of \( \omega_S = 6 \text{ rad/s} \), determine the angular velocity of link \( AB \) at this instant. The sprocket \( S \) is mounted on a shaft which is separate from the collinear shaft attached to \( AB \) at \( A \). The pin at \( C \) is attached to one of the chain links.

Kinematic Diagram: Since link \( AB \) is moving about the fixed point \( A \), then \( \omega_A \) is always directed perpendicular to link \( AB \) and its magnitude is \( \omega_A = \omega_S \). At the center shown, \( \omega_A \) is directed at an angle of \( 90^\circ \) with the horizontal. Since point \( C \) is attached to the chain, as shown in the above figure, it moves vertically with a speed of \( v_C = \omega_A \gamma = 6(0.175) = 1.05 \text{ m/s} \).

Kinematics Equation: The instantaneous center of zero velocity of link \( BC \) is on the center line drawn perpendicular to \( \omega_A \) and \( v_C \). Using law of sines, we have

\[ \begin{align*}
\text{Law of sines:} & \\
\frac{\omega_B}{\sin 105^\circ} &= \frac{\omega_C}{\sin 30^\circ} \\
\omega_B &= 0.286 \text{ rad/s}
\end{align*} \]

The angular velocity of link \( BC \) is given by

\[ \omega_C = \frac{\omega_A}{\sin 30^\circ} = 0.8 \text{ rad/s} \]

Thus, the angular velocity of link \( AB \) is given by

\[ \begin{align*}
\omega_A &= \omega_S - \omega_B - \omega_C \\
&= 6 - 0.8 - 0.286 \\
&= 4.914 \text{ rad/s}
\end{align*} \]

\[ \omega_A = 7.17 \text{ rad/s} \]
16-109. The wheel is moving to the right such that it has an angular velocity \( \omega = 2 \, \text{rad/s} \) and angular acceleration \( \alpha = 4 \, \text{rad/s}^2 \) at the instant shown. If the disk does not slip at \( A \), determine the acceleration of point \( B \).

Since no slipping:

\[
\begin{align*}
\alpha & = \omega' = 4 \, \text{rad/s}^2 \\
\omega & = \omega_0 + \alpha t = 2 \, \text{rad/s} + 4t \\
\end{align*}
\]

\[
\begin{align*}
\alpha & = \omega'' = \omega' + \alpha = 2 + 4t + 4t = 6t + 2 \\
\end{align*}
\]

\[
\begin{align*}
\theta & = \int \omega dt = \int (2 + 4t) dt = 2t + 2t^2 \\
\end{align*}
\]

\[
\begin{align*}
\theta & = \int \alpha dt = \int (6t + 2) dt = 3t^2 + 2t \\
\end{align*}
\]

\[
\begin{align*}
\vec{r}_B & = \vec{r}_A + \vec{r}_{AB} = (2 + 2t) \hat{i} + (3t^2 + 2t) \hat{j} \\
\end{align*}
\]

\[
\begin{align*}
\vec{a}_B & = \vec{a}_A + \vec{a}_{AB} = 6t + 2 \hat{i} + 6t \hat{j} \\
\end{align*}
\]

\[
\begin{align*}
\vec{a}_A & = \vec{a}' + \vec{a}_{AB} = \omega' \times \vec{r}_A + \vec{a}_{AB} = 6t \hat{i} + 2 \hat{j} \\
\end{align*}
\]

\[
\begin{align*}
\vec{a}_{AB} & = \vec{a}_B - \vec{a}_A = (6t + 2 - 6t) \hat{i} + (6t - 2) \hat{j} \\
\end{align*}
\]

\[
\begin{align*}
\vec{a}_{AB} & = 6t(\hat{i} - \hat{j}) \\
\end{align*}
\]

16-110. At a given instant the wheel is rotating with the angular motion shown. Determine the acceleration of the collar at \( A \) at this instant.

\[
\begin{align*}
\theta & = \theta_0 + \omega_0 t + \alpha_0 t^2 = 90 - 0.1t + 0.7t^2 \\
\omega & = \omega_0 + \alpha_0 t = -0.2 + 0.7t \\
\end{align*}
\]

\[
\begin{align*}
\alpha & = \omega' = 0.7 \, \text{rad/s}^2 \\
\end{align*}
\]

\[
\begin{align*}
\vec{r}_A & = (r \cos \theta) \hat{i} + (r \sin \theta) \hat{j} = (1.2 \cos (90 - 0.1t + 0.7t^2)) \hat{i} + (1.2 \sin (90 - 0.1t + 0.7t^2)) \hat{j} \\
\vec{v}_A & = \vec{v}' + \vec{v}_{AB} = \omega \times \vec{r}_A + \vec{v}_{AB} \\
\vec{a}_A & = \vec{a}' + \vec{a}_{AB} = \alpha \times \vec{r}_A + \vec{a}_{AB} \\
\end{align*}
\]

\[
\begin{align*}
\vec{a}_{AB} & = \vec{a}_A - \vec{v}_A = (\alpha \times \vec{r}_A - \omega \times \vec{r}_A) + \vec{a}_{AB} \\
\end{align*}
\]

\[
\begin{align*}
\vec{a}_{AB} & = (0.7 \times 1.2 \cos (90 - 0.1t + 0.7t^2)) \hat{i} - (0.7 \times 1.2 \sin (90 - 0.1t + 0.7t^2)) \hat{j} \\
\end{align*}
\]

\[
\begin{align*}
\vec{a}_{AB} & = (2.1 \cos (90 - 0.1t + 0.7t^2)) \hat{i} - (2.1 \sin (90 - 0.1t + 0.7t^2)) \hat{j} \\
\end{align*}
\]

\[
\begin{align*}
\vec{v}_A & = \vec{v}' + \vec{v}_{AB} = \omega \times \vec{r}_A + \vec{v}_{AB} \\
\end{align*}
\]

\[
\begin{align*}
\vec{v}_A & = (\omega \times \vec{r}_A - \omega \times \vec{r}_A) + \vec{v}_{AB} \\
\vec{v}_A & = (0.7 \times 1.2 \sin (90 - 0.1t + 0.7t^2)) \hat{i} - (0.7 \times 1.2 \cos (90 - 0.1t + 0.7t^2)) \hat{j} \\
\vec{v}_A & = (2.1 \sin (90 - 0.1t + 0.7t^2)) \hat{i} - (2.1 \cos (90 - 0.1t + 0.7t^2)) \hat{j} \\
\end{align*}
\]

\[
\begin{align*}
\vec{a}_{AB} & = (2.1 \cos (90 - 0.1t + 0.7t^2)) \hat{i} - (2.1 \sin (90 - 0.1t + 0.7t^2)) \hat{j} \\
\end{align*}
\]

\[
\begin{align*}
\vec{a}_{AB} & = (2.1 \times 1.2 \cos (90 - 0.1t + 0.7t^2)) \hat{i} - (2.1 \times 1.2 \sin (90 - 0.1t + 0.7t^2)) \hat{j} \\
\vec{a}_{AB} & = (2.52 \cos (90 - 0.1t + 0.7t^2)) \hat{i} - (2.52 \sin (90 - 0.1t + 0.7t^2)) \hat{j} \\
\vec{a}_{AB} & = (2.52 \cos (90 - 0.1t + 0.7t^2)) \hat{i} - (2.52 \sin (90 - 0.1t + 0.7t^2)) \hat{j} \\
\end{align*}
\]
16-115. The hoop is cast on the rough surface such that it has an angular velocity \( \omega = 4 \text{ rad/s} \) and an angular acceleration \( \alpha = 5 \text{ rad/s}^2 \). Also, its center has a velocity \( v_{c0} = 5 \text{ m/s} \) and a deceleration \( a_{c0} = 2 \text{ m/s}^2 \). Determine the acceleration of point A at this instant.

\[
\begin{align*}
\mathbf{a}_0 &= a_r + a_{c0} \\
\mathbf{a}_0 &= \left[ \begin{bmatrix} 2.5 \ \ 1 \ \ 5 \end{bmatrix} \right] + \left[ \begin{bmatrix} 10 \ \ 0 \ \ 31 \end{bmatrix} \right] \\
\mathbf{a}_0 &= \left[ \begin{bmatrix} 3.5 \ \ 1 \ \ 36.5 \end{bmatrix} \right] \\
a_r &= 5.04 \text{ m/s}^2 \\
\theta &= \tan^{-1} \left( \frac{4.38}{3.5} \right) = 53.9^\circ \quad \text{Ans}
\end{align*}
\]

Also:

\[
\begin{align*}
\mathbf{a}_0 &= a_r + a_{c0} \\
a_r &= -4 \omega^2 T_{c0} + \alpha \times T_{c0} \\
a_r &= -20 - (4 \times \omega^2 T_{c0} + 5 \mathbf{k} \times (0,0,0)) \\
a_r &= -3.5 - 4.8 \text{ m/s}^2 \\
a_r &= 5.04 \text{ m/s}^2 \\
\theta &= \tan^{-1} \left( \frac{4.38}{3.5} \right) = 53.9^\circ \quad \text{Ans}
\end{align*}
\]

16-116. The hoop is cast on the rough surface such that it has an angular velocity \( \omega = 4 \text{ rad/s} \) and an angular acceleration \( \alpha = 5 \text{ rad/s}^2 \). Also, its center has a velocity of \( v_{c0} = 5 \text{ m/s} \) and a deceleration \( a_{c0} = 2 \text{ m/s}^2 \). Determine the acceleration of point B at this instant.

\[
\begin{align*}
\mathbf{a}_0 &= a_r + a_{c0} \\
\mathbf{a}_0 &= \left[ \begin{bmatrix} 2.5 \ \ 0 \ \ 5 \end{bmatrix} \right] + \left[ \begin{bmatrix} 10 \ \ 0 \ \ 31 \end{bmatrix} \right] \\
\mathbf{a}_0 &= \left[ \begin{bmatrix} 3.5 \ \ 0 \ \ 36.5 \end{bmatrix} \right] \\
a_r &= 6.21 \text{ m/s}^2 \\
\theta &= \tan^{-1} \left( \frac{4.455}{4.335} \right) = 43.8^\circ \quad \text{Ans}
\end{align*}
\]

Also:

\[
\begin{align*}
\mathbf{a}_0 &= a_r + a_{c0} \\
a_r &= -4 \mathbf{k} \times T_{c0} - \omega^2 T_{c0} \\
a_r &= -70 \times \mathbf{k} \times (0.3 \cos 45^\circ - 0.3 \sin 45^\circ) = (42)(0.1 \cos 45^\circ - 0.1 \sin 45^\circ) \\
a_r &= -43.35 \text{ m/s}^2 \\
a_r &= 6.21 \text{ m/s}^2 \\
\theta &= \tan^{-1} \left( \frac{4.455}{4.335} \right) = 43.8^\circ \quad \text{Ans}
\end{align*}
\]
At a given instant, the gear has the angular motion shown. Determine the accelerations of point A and B on the link and the link’s angular acceleration at this instant.

For the gear:
\[ \alpha_t = \omega_{rev} = 6(12) = 72 \text{ in/a}^2 \]
\[ \alpha_t = -2(12)(-50) = 1200 \text{ in/a}^2 \]
\[ \alpha_t = \omega_0 + \alpha t = 72 + 6t \]
\[ \alpha_t = \frac{d}{dt} \left( \alpha_t \right) = 6 \text{ rad/a}^2 \]
\[ \theta = \alpha_0 t + \frac{1}{2} \alpha t^2 = 72t + 3t^2 \text{ rad} \]

For link AB:
The F/C is 0, so \( a_{AB} = 0 \), i.e.,
\[ \alpha_B = \frac{d}{dt} \left( \theta - \theta_0 \right) = 6 \]
\[ a_{BA} = -\omega_{AB} \times, \quad \omega_{AB} = (18000 \text{ rad} + 18000 \text{ rad}^3) \text{ rad} \]
\[ a_{BA} = \frac{d}{dt} \left( -\omega_{AB} \right) = -6 \text{ rad/a}^2 \]
\[ \alpha_B + \omega_{AB} \times = \frac{d}{dt} \left( \alpha_B + \omega_{AB} \right) = (18000 + 18000 \text{ rad}) \]
\[ \alpha_B = -12 \text{ rad/a} \]
\[ r_{BA} = 18 \text{ rad/a}^2 \]
16-133. The man stands on the platform at \( P \) and runs out toward the edge such that when he is at \( A \), \( y = 5 \text{ ft} \). His mass center has a velocity of 2 ft/s and an acceleration of 5 ft/s², both measured with respect to the platform and directed along the \( y \)-axis. If the platform has the angular motion shown, determine the velocity and acceleration of his mass center at this instant.

\[
\begin{align*}
v_y &= v_0 + (\alpha \times r)_{\text{cm}} + (\omega \times v)_{\text{cm}} \\
v_y &= 2 \text{ ft/s} \times (0.25 \text{ rad/s}) \times 5 \text{ ft} \\
v_y &= 2 \text{ ft/s} + 1.25 \text{ ft/s} = 3.25 \text{ ft/s} \\
\alpha &= 0 \text{ rad/s}^2 + 2 \text{ rad/s}^2 = 2 \text{ rad/s}^2 \\
a_y &= 0 \text{ ft/s}^2 + 5 \text{ ft/s}^2 = 5 \text{ ft/s}^2 \\
a_x &= 0 \text{ ft/s}^2 + 0 \text{ ft/s}^2 = 0 \text{ ft/s}^2 \\
a_x &= 2 \text{ ft/s}^2 - 5 \text{ ft/s}^2 = -3 \text{ ft/s}^2
\end{align*}
\]

\[\text{Ans}\]

16-134. Block \( B \) moves along the slot in the platform with a constant speed of 2 ft/s, measured relative to the platform in the direction shown. If the platform is rotating at a constant rate of \( \omega = 5 \text{ rad/s} \), determine the velocity and acceleration of the block at the instant \( \theta = 60^\circ \).

\[
\begin{align*}
t_{\text{cm}} &= \frac{2 \text{ ft}}{\cos 60^\circ} = 4 \text{ ft} \\
t_r &= v_0 + \omega \times r + (\omega \times v)_{\text{cm}} \\
t_r &= 2 \text{ ft/s} + (5 \text{ rad/s}) \times (4 \text{ ft}) \\
t_r &= 2 \text{ ft/s} + 20 \text{ ft/s} = 22 \text{ ft/s} \\
t_r &= (-12.01, 5.71) \text{ ft/s} \\
\alpha &= 0 \text{ rad/s}^2 + 2 \text{ rad/s}^2 = 2 \text{ rad/s}^2 \\
a_r &= 0 \text{ ft/s}^2 + 10 \text{ ft/s}^2 = 10 \text{ ft/s}^2 \\
a_r &= 0 \text{ ft/s}^2 + 20 \text{ ft/s}^2 = 20 \text{ ft/s}^2 \\
a_r &= 20 \text{ ft/s}^2 - 30 \text{ ft/s}^2 = 10 \text{ ft/s}^2
\end{align*}
\]

\[\text{Ans}\]
A girl stands at $A$ on a platform which is rotating with a constant angular velocity $\omega = 0.5 \text{ rad/s}$. If she walks at a constant speed of $v = 0.75 \text{ m/s}$ measured relative to the platform, determine her acceleration $(a)$ when she reaches point $D$ in going along the path $ABC$, $r = 1 \text{ m}$; and $(b)$ when she reaches point $B$ if she follows the path $ABC$, $r = 3 \text{ m}$.

\[ a_y = a_y + \dot{\omega} \times r_y + \dot{r} \times (\dot{r} \times r_y) + 2 \ddot{r} \times (\dot{r} \times r_y) + \dddot{r} \times (r_y + \dot{r} \times r_y) \]

\[ \dot{\omega} = 0 \text{ rad/s} \]

\[ \dot{r} = (0.75) \text{ m/s} \]

\[ \ddot{r} = 0 \text{ m/s}^2 \]

Substitute the data into Eq.(1):

\[ a_y = 0 + (0.75)(0.75) + (0.75)(0.75) + 0 = 1.125 \text{ m/s}^2 \]

\[ \omega = \sqrt{\ddot{r} \times (\dot{r} \times r_y)} = \sqrt{(0.75)^2} = 0.75 \text{ rad/s} \]

\[ r = 0 \text{ m} \]

\[ \dot{r} = (0.75) \text{ m/s} \]

\[ \ddot{r} = 0 \text{ m/s}^2 \]

Substitute the data into Eq.(2):

\[ a_y = 0 + (0.75)(0.75) + (0.75)(0.75) + 0 = 1.125 \text{ m/s}^2 \]
Rod $AB$ rotates counterclockwise with a constant angular velocity $\omega = 3 \text{ rad/s}$. Determine the velocity and acceleration of point $C$ located on the double collar when $\theta = 45^\circ$. The collar consists of two pin-connected slider blocks which are constrained to move along the circular path and the rod $AB$.

$$v_C = (0.40m + 0.40m)$$
$$v_C = -v_A$$
$$v_C = v_x + \Omega \times r_{CA}$$
$$v_C = 0 + (0.40m + 0.40m) + (v_x + 0.40m + 0.40m)$$
$$2v_x = 0 - 1.2m + 1.2m = 0.20\text{m/s}$$
$$v_x = 0.10\text{m/s}$$
$$v_y = -1.20 + 0.70\text{m/s}$$
$$v_y = 0.50\text{m/s}$$
$$v_y = 2.40 \text{m/s}$$
$$a_y = 1.87m/s^2$$
$$a_y = 2v_x + \Omega \times r_{CA} + \Omega \times (\Omega \times r_{CA}) + \Omega \times (\Omega \times r_{CA})$$
$$a_y = 0 + 0 + 30 + 0.20 + 0.12\text{m/s}^2$$
$$a_y = 3.32\text{m/s}^2$$
$$a_y = 3.32 + 3.20 = 6.52\text{m/s}^2$$
$$a_y = 6.52 = 380 + 0.70\text{m/s}^2$$
$$a_y = 6.52 = 3.80 + 1.20 + 0.70\text{m/s}^2$$
$$v_{CA} = 2.00\text{m/s}$$
$$a_{CA} = 0$$

Thus,
$$a_y = a_{CA} = 1.87\text{m/s}^2$$
$$a_y = 1.87 \text{m/s}^2$$
$$a_y = (-14.44) \text{m/s}^2$$