14-6. When a 7-kg projectile is fired from a cannon barrel that has a length of 2 m, the explosive force exerted on the projectile, while it is in the barrel, varies in the manner shown. Determine the approximate muzzle velocity of the projectile at the instant it leaves the barrel. Neglect the effects of friction inside the barrel and assume the barrel is horizontal.

The work done is measured as the area under the force—displacement curve. This area is approximately 31.8 square meters. Since each square has an area of 12.5 mm$^2$, we have

\[ F(t) \cdot d(t) = 12.5 \text{ mm}^2 \]

\[ \sum F_{m} = F(t) 

\[ 0 + \int_{0}^{t} \frac{1}{2}(12.5) dt = 12.5 \text{ mm}^2 \]

\[ v_m = \frac{31.8}{12.5} = 2.54 \text{ m/s} \] (approx.)

Ans

14-7. Design considerations for the bumper $B$ on the 5-Mg train car require use of a nonlinear spring having the load-deflection characteristics shown in the graph. Select the proper value of $k$ so that the maximum deflection of the spring is limited to 0.2 m when the car, traveling at 4 m/s, strikes the rigid stop. Neglect the mass of the car wheels.

\[ s = \frac{5000k}{2} \left( \frac{v}{2} \right)^2 \]

\[ s = \frac{5000k}{2} \times \left( \frac{4}{2} \right)^2 \]

\[ s = 50 \text{ m} \]

\[ k = 150 \text{ Mm}^2 \text{N} \]

Ans

14-8. The crate, which has a mass of 100 kg, is subjected to the action of the two forces. If it is originally at rest, determine the distance it slides in order to attain a speed of 6 m/s. The coefficient of kinetic friction between the crate and the surface is $\mu_k = 0.2$.

\[ \sum F_{x} \text{ at rest: } F_1 = \mu_k F_N \]

\[ F_1 = \mu_k F_N \]

\[ N = 100 \times 0.2 = 20 \text{ N} \]

\[ F_1 = 0.2 \times 20 = 4 \text{ N} \]

\[ N = F_1 \]

\[ N = 100 \text{ N} \]

\[ F_2 = 100 \text{ N} \]

\[ F_1 + \sum U_{x} = F_2 \]

\[ 0 + 100 \cos 30^\circ \times 10 + 100 \times \left( \frac{4}{3} \right)^2 = 100 \times 0.6^2 \]

\[ s = 3.54 \text{ m} \]

Ans
14.14. Determine the velocity of the 20-kg block A after it is released from rest and moves 2 m down the plane. Block B has a mass of 10 kg and the coefficient of kinetic friction between the plane and block A is \( \mu_k = 0.2 \). Also, what is the tension in the cord?

**Block A**

\[ N_x A \cos 60^\circ = 0 \]

\[ N_x = 98.1 \text{ N} \]

**System**

\[ T \sin \theta = \mu_k N_x \]

\[ \begin{align*}
T &\cdot 0.25 \times 1000 \text{ N/s} = 0.2 \times 98.1 \times 1000 \text{ N/s} + \frac{1}{2} (200)^2 + \frac{1}{2} (100)^2 \\
T &\approx 2304 \text{ N}
\end{align*} \]

**Ans**

Also, block A:

\[ T \sin \theta = \mu_k N_x \]

\[ 2304 \text{ N} \]

**Ans**

**Block B**

\[ T \sin \theta = \mu_k N_x \]

\[ 0.2 \times 98.1 \times 1000 \text{ N/s} = \frac{1}{2} (100)^2 + \frac{1}{2} (100)^2 \\
T \approx 115 \text{ N} \]

**Ans**

14.15. Block A has a weight of 60 lb and block B has a weight of 10 lb. Determine the speed of block A after it moves 5 ft down the plane, starting from rest. Neglect friction and the mass of the cord and pulleys.

\[ \begin{align*}
T \sin \theta &= \mu_k N_x \\
200 \sin \theta &= 0.2 \times 98.1 \times 1000 \\
200 \sin \theta &= 0 \\
N_x &= 118.6 \text{ lb} \\
\end{align*} \]

\[ \begin{align*}
2v_x + 4v_y &= 0 \\
2v_x &= v_y = 0 \\
N_x &= 118.6 \text{ lb} \\
0 &= 400 \sin \theta - 10 \times 9.81 \\
&= 400 \times 0.25 - 10 \times 9.81 \\
&= 100 - 98.1 \\
&= 1.9 \\
&= 1.9 \text{ ft/s} \\
\end{align*} \]

\[ \text{Ans} \]
14-29. Roller coasters are designed so that riders will not experience more than 3.5 times their weight as a normal force against the seat of the car. Determine the smallest radius of curvature \( r \) of the track at its lowest point if the car has a speed of 50 ft/s at the crest of the drop. Neglect friction.

**Principle of Work and Energy:** Here, the rider is taken to be ideal (no friction) and the car is considered to be a point mass. Applying Eq. 14-7, we have

\[
\sum \mathbf{F} = \frac{\Delta E}{\Delta t}
\]

\[
W = \mathbf{F}_n \cdot \mathbf{r} = \frac{1}{2} m v^2
\]

**Equation of Motion:** It is required that \( N = 3.5W \). Applying Eq. 13-7, we have

\[
N = m g = \frac{W}{\cos \theta} = \frac{1}{2} m v^2
\]

\[
\cos \theta = \frac{1}{2} \frac{v^2}{g}
\]

\[
\theta = \cos^{-1} \left( \frac{1}{2} \frac{v^2}{g} \right)
\]

\[
r = \frac{\Delta E}{\Delta t} = \frac{1}{2} m v^2
\]

\[
\frac{1}{2} m v^2 = mg \cos \theta
\]

\[
r = \frac{m g}{\frac{1}{2} m v^2}
\]

\[
r = 2g
\]

**Ans:**

\[r = 2g \approx 20 \text{ ft}
\]

14-30. A catapulting mechanism is used to propel the 10 kg slider \( A \) to the right along the smooth track. The propelling action is obtained by drawing the pulley attached to rod \( BC \) rapidly to the left by means of a piston \( F \). If the piston applies a constant force \( F = 20 \text{ kN} \) to rod \( BC \), then it moves at 0.2 m, determine the speed \( v \) at which the slider \( A \) was originally at rest. Neglect the mass of the pulleys, cable, piston, and rod \( BC \).

\[
F = m a = \frac{W}{g}
\]

\[
F = m g
\]

\[
F = m g
\]

\[
F = m g = \frac{1}{2} m v^2
\]

\[
0 = \left( \frac{100000 \times 2}{100000} \right) \times \left( \frac{1}{2} \times 10 \times 10 \right)
\]

\[
v = 25.3 \text{ m/s}
\]

**Ans:**

\[v = 25.3 \text{ m/s}
\]
14.31. The collar has a mass of 3 kg and slides along the uniform rod. Two springs are attached to it and the ends of the rod as shown. Each spring has an unextended length of 1 m and the collar has a speed of 2 m/s when \( s = 0 \). Determine the maximum compression of each spring due to the back and forth (oscillating) motion of the collar.

\[
E_1 + W_{p_1} = E_2 \\
\frac{1}{2}m(2s)^2 + \frac{1}{2}k(s - 0.25)^2 - \frac{1}{2}m(2s)^2 = 0
\]

\[ s = 0.756 \text{ m} \] 

---

14.32. The cyclist travels to point A, pedaling until he reaches a speed \( v_A = 8 \text{ m/s} \). He then coast freely up the curved surface. Determine the normal force he exerts on the surface when he reaches point B. The total mass of the bike and mass is 75 kg. Neglect friction, the mass of the wheels, and the size of the bicycle.

\[
\frac{1}{2}m^2 \frac{dv}{dt} = -g
\]

\[
t = \frac{1}{2}a \frac{d^2 y}{dt^2} = \frac{1}{2}
\]

\[
x = \frac{y}{2} - a
\]

\[ x = 2y - \frac{1}{2}a^2 \\
\]

\[ 2y = -1 \\
\]

\[ a = 2 \text{ m/s}^2 \\
\]

\[ 1 = \frac{1}{2} \left( 2 \cdot \frac{dy}{dt} \right) \\
\]

\[ y = \frac{1}{2} - \frac{1}{2} \left( \frac{dy}{dt} \right)^2 \\
\]

\[ x = \frac{1}{2} - \frac{1}{2} \left( \frac{dy}{dt} \right)^2 \\
\]

\[ 2x + 1 = 1 \\
\]

\[ \frac{dy}{dt} = 1 \\
\]

\[ x = \frac{1}{2} - \frac{1}{2} \left( \frac{dy}{dt} \right)^2 = 1 \\
\]

\[ v = \frac{dx}{dt} = 2 \text{ m/s} \\
\]

\[ y = t \\
\]

\[ x = \frac{1}{2} + t \\
\]

\[ x = \frac{1}{2} + \frac{1}{2} t \\
\]

\[ x = \frac{1}{2} + \frac{1}{2} t \\
\]

\[ t = \text{ Answer} \\
\]

\[ \text{Answer:} \] 

\[ x = 1.5 \text{ m} \\
\]

\[ E = E_1 + \frac{1}{2}mv^2 = \frac{1}{2}m(2s)^2 + \frac{1}{2}k(s - 0.25)^2 + \frac{1}{2}m(2s)^2 = 0 \]

\[ s = 0.756 \text{ m} \]
14-45. An automobile having a mass of 2 Mg travels up a 7° slope at a constant speed of \( v = 100 \text{ km/h} \). If mechanical friction and wind resistance are neglected, overcome the power developed by the engine if the automobile has an efficiency \( \eta = 0.65 \).

\[
\sum F = ma_v; \quad F = \frac{100(10^3) \text{ m}}{1 \text{ h}} \cdot \frac{3.75 \text{ km}}{500 \text{ m}} \cdot \frac{0.65}{27.8 \text{ m/s}} = 2391 \text{ N}
\]

Power: Here, the speed of the car is \( v = \frac{100(10^3) \text{ m}}{1 \text{ h}} \cdot \frac{3.75 \text{ km}}{500 \text{ m}} \cdot \frac{0.65}{27.8 \text{ m/s}} = 2391 \text{ N} \).

\[
P = F \cdot v = 2391 \text{ N} \cdot 27.8 \text{ m/s} = 66418 \text{ N} \cdot \text{m/s} = 66418 \text{ W}
\]

Using Eq. 14.11, the required power input from the engine to provide the above power output is
\[
power \ input = \frac{power \ output}{\eta} = \frac{66418 \text{ W}}{0.65} = 102 \text{ kW}
\]

14-46. A loaded truck weighs 16 (15\(^3\)) lb and accelerates uniformly on a level road from 15 ft/s to 30 ft/s during 4 s. If the frictional resistance to motion is 325 lb, determine the maximum power that must be delivered to the wheels.

\[
\frac{\Delta v}{\Delta t} = \frac{30 - 15}{4} = 3.75 \text{ ft/s}^2
\]

\[
\sum F = ma_v; \quad F = 325 \left( \frac{30(15^3)}{32.5} \right) (3.75) = 2188.3 \text{ lb}
\]

\[
P_{\text{max}} = \frac{2188.3 \times 30}{500} = 119 \text{ W}
\]

\[
power \ input = 119 \text{ W}
\]

\[
= 175 \text{ lb}
\]
14-51. To demonstrate the loss of energy in an automobile, consider a car having a weight of 5000 lb that is traveling at 75 mph. If the car is brought to a stop, determine how long a 100-W light bulb must burn to expend the same amount of energy. (1 hp = 528.0 ft-lb)

Range: How, the speed of the car is

\[
\frac{300}{44} = 6.8 \text{ sec.}
\]

The power of the bulb is

\[
P = 100 \frac{528}{1} = 52800 \text{ ft-lb per sec.}
\]

The time is

\[
\frac{52800}{100} = 528 \text{ sec.}
\]

Therefore, the bulb must burn for 528 sec.

14-52. The motor M is used to hoist the 500-kg elevator upward with a constant velocity \( v_2 = 2 \text{ m/s} \). If the motor draws 50 kW of electrical power, determine the motor's efficiency. Neglect the mass of the pulleys and cable.

\[
F = mg = 500 \times 9.8 = 4900 \text{ N}
\]

\[
2F = 2 	imes 4900 = 9800 \text{ N}
\]

\[
T_E = F - 2F = 4900 \text{ N}
\]

\[
T = 50 \times 10^3 \text{ W}
\]

\[
\eta = \frac{T_E}{T} = \frac{4900}{50 \times 10^3} = 0.98
\]

14-53. The 500-kg elevator starts from rest and travels upward with a constant acceleration \( a = 2 \text{ m/s}^2 \). Determine the power output of the motor M when \( v = 3 \text{ m/s} \). Neglect the mass of the pulleys and cable.

\[
\frac{1}{2}m v^2 = m a t^2
\]

\[
t = \sqrt{\frac{2m a}{m v^2}} = \sqrt{\frac{2 	imes 500 \times 2}{500 	imes 9.8}} = 0.7 \text{ s}
\]

\[
F = \frac{ma}{t} = \frac{500 \times 2}{0.7} = 1428.57 \text{ N}
\]

\[
T = \frac{Fv}{2} = \frac{1428.57 \times 3}{2} = 2142.86 \text{ W}
\]
14-56. The 50 kg crate is hoisted up the 30° incline by the pulley system and motor M. If the crate starts from rest and its constant acceleration attains a speed of 4 m/s after traveling 8 m along the plane, determine the power that must be supplied to the motor at this instant. Neglect friction along the plane. The motor has an efficiency of \(\eta = 0.54\).

\[s = \frac{1}{2}at^2 = 2 \times 10^{-2} (2t - t^2)\]
\[v = at = 4t\]
\[F = \frac{m \cdot \Delta v}{\Delta t} = 100 \text{ N}\]
\[p = F \cdot v = 400 \times 4 = 1600 \text{ W}\]

14-57. The sports car has a mass of 2.3 Mg and, while it is traveling at 28 m/s, the driver causes it to accelerate at 5 m/s². If the drag resistance on the car due to the wind is \(F_D = 0.5 \text{ N/m/s}^2\), where \(v\) is the velocity in m/s, determine the power supplied to the engine at this instant. The engine has a running efficiency of \(\eta = 0.60\).

\[\frac{m \cdot \Delta v}{\Delta t} = 23 \text{ Nm/s}^2\]
\[\eta \cdot P = \frac{N \cdot V}{W} = 28 \text{ kW}\]

14-58. The sports car has a mass of 2.3 Mg and accelerates at 6 m/s², starting from rest. If the drag resistance on the car due to the wind is \(F_D = 100 \text{ N}\), where \(v\) is the velocity in m/s, determine the power supplied to the engine when \(t = 5 \text{ s}\). The engine has a running efficiency of \(\eta = 0.60\).

\[\frac{m \cdot \Delta v}{\Delta t} = 23 \text{ Nm/s}^2\]
\[\eta \cdot P = \frac{N \cdot V}{W} = 30 \text{ kW}\]
14-74. The collar has a weight of 8 lb. If it is released from rest at a height of \( h = 2 \) ft from the top of the uncompressed spring, determine the speed of the collar after it falls and compresses the spring 0.5 ft.

14-75. The 2-kg collar is attached to a spring that has an unstretched length of 3 m. If the collar is drawn to point B and released from rest, determine its speed when it arrives at point A.

**Potential Energy:** The initial and final elastic potential energies are

\[
\frac{1}{2}k(3.5)^2 = 6.0 \text{ J} \quad \text{and} \quad \frac{1}{2}k(2.75)^2 = 4.1 \text{ J},
\]

respectively. The gravitational potential energy remains the same case the elevation of collar does not change when it moves from B to A.

**Conservation of Energy:**

\[
E + V_B = E + V_A
\]

\[
0 + 6.0 = 4.1 + \frac{1}{2}mV_A^2
\]

\[
V_A = 2.45 \text{ m/s Answer}
\]

14-76. The 5-lb collar is released from rest at A and travels along the smooth guide. Determine the speed of the collar just before it strikes the stop at B. The spring has an unstretched length of 12 in.

**Conservation of Energy:**

\[
E + V_B = E + V_A
\]

\[
0 + \frac{1}{2}mV_B^2 = \frac{1}{2}kx^2 + \frac{1}{2}\left(\frac{1}{2}mV_A^2 - \frac{1}{2}kx^2\right)
\]

\[
V_A = 15.0 \text{ ft/s Answer}
\]

14-77. The 5-lb collar is released from rest at A and travels along the smooth guide. Determine its speed when it reaches point C and the normal force it exerts on the rod at this point. The spring has an unstretched length of 12 in., and point C is located just before the end of the curved portion of the rod.

**Conservation of Energy:**

\[
E + V_B = E + V_A
\]

\[
0 + \frac{1}{2}kx^2 + \frac{1}{2}kx^2 = \frac{1}{2}V_B^2
\]

\[
x = 12 \text{ in.}
\]

\[
V = 12.5 \text{ ft/s Answer}
\]

**Normal Force:**

\[
F_N = mV + \frac{1}{2}kx^2 = 5 \times 12.5 + \frac{1}{2} \times 5 \times (12)^2 = 155 \text{ lb}
\]

Thus,

\[
N_x = 15.5 \text{ lb Answer}
\]
The roller-coaster car has a mass of 800 kg, including its passengers. If it is released from rest at the top of the hill, determine the minimum height $h$ of the hill so that the car travels around both inside loops without leaving the track. Neglect friction, the mass of the wheels, and the size of the car. What is the normal reaction on the car when the car is at $A$ and when it is at $C$?

$$0 = \frac{1}{2}mv^2 = \frac{1}{2}(800 \text{ kg})(14.0 \text{ m/s})^2$$

$$h = \frac{\text{KE}}{m} = \frac{1}{2}(800 \text{ kg})(14.0 \text{ m/s})^2$$

$$h = 560 \text{ m}$$

At $A$:

$$N_A = 800 \text{ kg} \times 14.0 \text{ m/s}^2$$

At $C$:

$$N_C = 0$$

Since friction is negligible, the car will speed around the 7 m loop provided it has speed $14.0 \text{ m/s}$.
14-81. Treeam has a mass of 100 kg and from it swings from the cliff by rigidly holding on to the tree vine, which is 10 m measured from the supporting limb A to his center of mass. Determine the speed just after the vine strikes the lower limb at B. Also, with what force must he hold on to the vine just before and just after the vine contacts the limb at B?

\[ T + \ell = 5 \times 10 \text{ m} \]

\[ 0 + \frac{1}{2}(100)(v)^2 = 1000 \times 9.81 \times 10 \times (1 - \cos 45°) \]

\[ v = 7.58 \text{ m/s} \] \text{Ans}

Just before striking B, \( \mu = 10 \text{ m} \)

\[ T = \frac{7.58^2}{10} \]

\[ T = 1.54 \text{ kN} \] \text{Ans}

Just after striking B, \( \mu = 10 \text{ m} \)

\[ T = \frac{7.58^2}{3} \]

\[ T = 2.56 \text{ kN} \] \text{Ans}

14-82. The spring has a stiffness \( k = 3 \text{ kN/m} \) and an unstretched length of 2 m. If it is attached to the 4 th support collar and the collar is released from rest at A, determine the speed of the collar just before it strikes the end of the rod at B. Neglect the size of the collar.

\[ k = 3 \text{ kN/m} \]

\[ l_{oa} = \sqrt{(1.2)^2 + (0.4)^2} = 1.28 \text{ m} \]

\[ l_{oa} = \sqrt{(1.1)^2 + (0.3)^2} = 1.24 \text{ m} \]

\[ V_a = V_i = \frac{1}{2} \text{ m/s} \]

\[ 0 + \frac{1}{2}(3)(2.58 - 2)^2 = \frac{1}{2}(3)(1.28 - 2)^2 \]

\[ V_a = 272 \text{ m/s} \] \text{Ans}
34-86. When the 4-kg box reaches point A it has a speed of \( v_A = 2 \text{ m/s} \). Determine the angle \( \theta \) at which it leaves the smooth circular ramp and the distance \( x \) to where it falls into the cart. Neglect friction.

As per Eq. 8:

\[
g_E = mg = 60.81 \text{ N} = g \left( \frac{4}{2.17} \right)
\]

Choose a suitable coordinate system:

\[
\begin{align*}
\v_0 &= \begin{pmatrix} x_0 \ y_0 \ z_0 \ \theta \ \phi \end{pmatrix} \\
\v_t &= \begin{pmatrix} x_t \ y_t \ z_t \ \phi_t \ \theta_t \end{pmatrix}
\end{align*}
\]

Substitute Eq. (1) into Eq. (2), and solving for \( \v_0 \):

\[
v_0 = 1.951 \text{ m/s}
\]

Then:

\[
\theta = \cos^{-1} \left( \frac{\sqrt{0.305}}{1.951} \right) = 25.88^\circ
\]

\[
\phi = 20^\circ = 27.3^\circ
\]

\[
(+) \quad x = x_0 + v_0 \cos \theta t
\]

\[
2.016 = (0.698 \cos 25.88^\circ t) + \frac{1}{2} (9.81 \sin 25.88^\circ) t^2
\]

Solving for the positive root:

\[
x = 0.204 \text{ s}
\]

\[
(+) \quad \theta = \cos^{-1} \left( \frac{\sqrt{0.305}}{1.951} \right)
\]

\[
\phi = 20^\circ = 27.3^\circ
\]

Distance from the point to:

\[
x = 0 \times (0.698 \cos 25.88^\circ) = 0.0387 \text{ m}
\]

\[
y = 0.387 \text{ m}
\]

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