Exercise 1. You are given vectors $\vec{a} = \langle 1, -2, 2 \rangle$ and $\vec{b} = \langle 2, 0, -3 \rangle$. For what value of $t$ does the vector $\vec{a} + t\vec{b}$ lie in the plane determined by the vectors $\vec{p} = \langle 1, 1, 1 \rangle$ and $\vec{q} = \langle 1, 2, 3 \rangle$?

**Hint:** Try to use triple scalar product. The final answer is $t = 7$.

Exercise 2. Use properties of the cross product and triple scalar product to simplify the following expressions:

(i) $(\vec{a} \times \vec{b}) \times (\vec{b} \times \vec{a})$ 
(ii) $(\vec{a} \times \vec{b}) \cdot \vec{a}$

Exercise 3. You are given the points $A(1, 5, -1)$, $B(3, 2, -2)$ and $C(1, 3, a)$, where $a$ is a real number. Let $l$ be the line determined by $A$ and the vector $\vec{s} = \langle 1, -1, 3 \rangle$. Find the value of $a$ so that the plane determined by $A$, $B$ and $C$ contains the line $l$.

**Answer:** $a = -15$.

Exercise 4. Consider the line $l : \frac{x - 4}{3} = \frac{y + 3}{-4} = \frac{z - 3}{2}$.

Find an equation of the line that passes through the point $(3, 2, 0)$ and is perpendicular to $l$.

**Hint:** First, find an equation of the plane containing the point $(3, 2, 0)$ and is perpendicular to $l$. Use this plane to get the second point on the line perpendicular to $l$. The final answer is $\frac{x - 3}{4} = \frac{y - 2}{9} = \frac{z - 0}{8}$.

Exercise 5. Let $l_1$ be defined as the line of intersection of the planes $y = 3$ and $2x + y - z = 6$. Additionally, let the line $l_2$ be defined as $l_2 : \frac{x - 1}{2} = \frac{y - 1}{2} = \frac{z - 2}{1}$.

Show that the lines $l_1$ and $l_2$ belong to the same plane.

**Hint:** You need to show that $l_1$ and $l_2$ intersect at some point or that $l_1$ and $l_2$ are parallel. In other words, you need to show that $l_1$ and $l_2$ are not skew lines.
Exercise 6. You are given the planes
\[ \pi_1 : 2x - y + z = 7, \quad \pi_2 : x - y = 4. \]
Let \( m \) be the line defined as the intersection of the planes \( \pi_1 \) and \( \pi_2 \).
You are also given the line
\[ l : \frac{x - 2}{0} = \frac{y + 1}{1} = \frac{z + 1}{4}. \]
Let \( T_1 \) and \( T_2 \) be the points of intersection of \( l \) with the planes \( \pi_1 \) and \( \pi_2 \) respectively. Find the projections of \( T_1 \) and \( T_2 \) onto \( m \).

Hint: First, you need to find \( T_1 \) and \( T_2 \). You should get \( T_1(2, 0, 3) \) and \( T_2(2, -2, 5) \). Then, you need to find the line \( m \), and you will get \( m : \frac{x - 4}{1} = \frac{y - 0}{1} = \frac{z + 1}{4} \). The projection of \( T_1 \) onto \( m \) is the intersection of \( m \) and the plane through \( T_1 \) perpendicular to \( m \). The same holds for \( T_2 \). The final answer is as follows: the projections of \( T_1 \) and \( T_2 \) onto \( m \) are \((2, -2, 1)\) and \((4, 0, -1)\) respectively.

Exercise 7. Consider the plane \( \pi : x - 3y + 2z + 5 = 0 \) and the sphere of radius 12 centered at \((5, -14, 9)\).

(i) Find the point \( P \) on \( \pi \) with the smallest distance from the sphere.
(ii) How far is \( P \) from the center of the sphere?
(iii) How far is \( P \) from the sphere?

Answer: (i) \( P(0, 1, -1) \); (ii) \( 5 \sqrt{14} \); (iii) \( 5 \sqrt{14} - 12 \).

Exercise 8. Let \( S \) be the sphere with the following properties: the radius of \( S \) is 3, the sphere \( S \) touches the plane \( 6x + 3y + 6z = 9 \) at the point \((1, -1, 1)\), and \( S \) is located in the half-space \( 6x + 3y + 6z \geq 9 \).

(i) Write an equation of the sphere \( S \).
(ii) Find all intersection points of the sphere \( S \) with the plane \( y = 3 \).

Answer: (i) \( (x - 3)^2 + y^2 + (z - 3)^2 = 9 \); (ii) point \((3, 3, 3)\).