

## MAP2302-LAB2/SUM09

1. Paul and Bob both decide to open small stores selling compact discs. “Paul’s Rock and Roll CD’S” AND “Bob’s Opera Only Discs” are located on the same block, and Paul and Bob soon become concerned about the effect of each store on the other. On the other hand, having two CD store on the same block might help both stores by attracting customer to the neighborhood. On the other hand the store may compete with each other for limited supply of customers. Paul and Bob argue about this until they are so sick of arguing that they hire the famous mathematician Glen to settle the matter. Glen follows his own advice about keeping the matter as simple as possible and suggests the following the system.

Let  $x(t)$  = daily profit’s of Paul’s store  $t$ .

That is if  $x(t) > 0$ , then Paul’s store is making money, but if  $x(t) < 0$  then Paul’s store loosing money. Since there is no hard information about how the profits of each store affect the change in profit of the other, Glen formulates the simplest possible model that allows each store to affect the other -a linear model. The systems is

$$\begin{aligned}\frac{dx}{dt} &= ax + by \\ \frac{dy}{dt} &= cx + dy\end{aligned}$$

where the  $a, b, c$ , and  $d$  are parameters. The rate of change of Paul’s profits depends linearly on both Paul’s and Bob’s profits(nothing else). The same assumptions apply for the Bob’s profits.

- (a) plot the phase portrait assuming  $a = d = 0, b = 1$ , and  $c = -1$  ; and interpret the result.
- (b) plot the phase portrait  $x(t)$  and  $y(t)$  graph of solution assuming  $a = 2, b = 1, c = -4$  and  $d = -1$ ; and interpret the result.