Problem 31-9 LC

$L = 50 \times 10^{-3} \text{ H} \quad C = 4 \times 10^{-6} \text{ F}

Current is initially $\max$.

(a) For $C$ to fully charge?

\[ i = -\omega A \sin (\omega t + \phi) \]

\[ t = 0 \quad i = -\omega A \sin \phi \quad \therefore \phi = -\frac{3\pi}{2} \text{ or } \frac{\pi}{2} \]

\[ q = A \cos (\omega t + \phi) \]

To fully charge need $\omega t^* + \phi = \pi$.

\[ t^* = \frac{\pi}{\omega} = \frac{\pi}{2 \sqrt{\frac{L}{2C}}} = \frac{\pi}{\sqrt{\frac{50 \times 10^{-3} \text{ H} \times 4 \times 10^{-6} \text{ F}}{2}}} \]

\[ t^* = 7 \times 10^{-4} \text{ s} \]
Problem 31-11 LC

\[ \begin{align*}
R &= 11 \, \Omega \\
C &= 6.2 \times 10^{-6} \, \text{F} \\
L &= 54 \times 10^{-3} \, \text{H} \\
\varepsilon &= 3 \, \text{V}
\end{align*} \]

\[ \text{a) for long time} \]

\[ \therefore \text{C is fully charged} \]

\[ i = 0 \]

\[ V = \frac{\varepsilon}{2} \Rightarrow q = EC \]

\[ \text{Throw to b} \]

\[ a) \ f \]

\[ \omega = 2\pi f = \frac{1}{\sqrt{LC}} \Rightarrow f = \frac{1}{2\pi} \frac{1}{\sqrt{LC}} = \frac{1}{2\pi \sqrt{54 \times 10^{-3} \times 6.2 \times 10^{-6}}} \]

\[ \omega = 1,728 \, \text{Hz} \Rightarrow f = 275 \, \text{Hz} \]

\[ b) \ \varphi(t) = A \cos(\omega t + \phi) \]

\[ \varphi(0) = EC = A \cos \phi = A \sin \phi + \phi = 0 \]

\[ i(t) = -\omega A \sin(\omega t + \phi) \]

\[ i_{\text{max}} = \omega A = \omega EC = 1,728 (34) 6.2 \times 10^{-6} \]

\[ i_{\text{max}} = 0.364 \, \text{Amp} \]
Problem 31-21. LC

\[ L = 3 \times 10^3 \text{H} \quad C = 2.7 \times 10^{-6} \text{F} \]

\[ t = 0 \quad i(0) = 0 = A \cos(\omega t + \phi) \]

\[ \therefore \quad q_1 = \frac{\pi}{2} \quad i = -wA \sin(\omega t + \pi/2) \]

\[ i(0) = 2 \text{Amps} \]

a) \[ i(t) = 2 \text{Amps} = -wA \sin \frac{\pi}{2} \]

\[ A = \frac{2}{\omega} = \frac{2}{2\pi} \sqrt{\frac{L}{C}} = 2\sqrt{(3 \times 2.7 \times 10^{-9}) \frac{1}{1}} \]

\[ q_{max} = \frac{A}{C} = 1.8 \times 10^{-4} \text{C} \]

b) \[ t > 0 \Rightarrow \frac{dq(t)}{dt} = i \text{ max} \]

\[ C = \frac{1}{\omega} \quad U_c = \frac{1}{2} CV_c^2 = \frac{q^2}{2C} \]

\[ \frac{dU_c}{dt} = \frac{1}{C} \frac{d}{dt} = \frac{A}{C} \cos(\omega t + \pi/2) (-\omega A) \sin(\omega t + \pi/2) \]

\[ \therefore \text{ and:} \quad \sin \alpha \cos \alpha = \frac{1}{2} \sin 2\alpha \quad \text{App Eq.} \]

\[ \frac{dU_c}{dt} = -\frac{\omega A^2}{C} - \frac{1}{2} \sin(2\omega t + \pi) \]

\[ \text{The max. when } \sin(\pi) = 1 = \sin(2\omega t + \pi) \]

\[ 2\omega t + \pi = 3\pi k \quad \text{or } t = \frac{\pi}{2\omega} = \frac{T}{4} \]

\[ t = \frac{\pi}{4} = 0.9 \times 10^{-4} = 7.07 \times 10^{-5} \]

\[ c) \quad \frac{dU_c}{dt} = -\frac{\omega A^2}{2C} \]

\[ = \frac{2L}{C} \quad U_c = -\frac{2L}{C} \quad \text{or } U_c = 2L \sqrt{\frac{1}{C}} = 2\sqrt{\frac{L}{C}} = \frac{C}{67\text{Watt}} \]
Problem 31-29: Simple AC Circuits \( L + C \)

(a) \( f = \frac{1}{\omega L} \) \( \omega = 6 \times 10^3 \) \( f = \frac{1}{2\pi \sqrt{LC}} \) or \( f = \frac{1}{\pi \sqrt{LC}} \)

\[
f = \frac{1}{2\pi} \times \frac{1}{\sqrt{6 \times 10^{-5}}} = \frac{650}{12} \text{ Hz}
\]

(b) \( X_L = \omega L = 2\pi f L = 2\pi (650) 6 \times 10^{-3} = 0.42 \text{ ohms} \)

(c) \( \omega = \frac{1}{\sqrt{LC}} = 2\pi f \) \( f = \frac{1}{2\pi \sqrt{LC}} \) same as above
Problem 31-31  Simple AC circuit.

\[ L = 5 \times 10^3 \text{H} \]

\[ E_m = 30 \text{V} \]

\[ v(t) = E_m \sin \omega t \]

**Current amplitude?**

\[ i(t) = \frac{E_m}{\omega L} \text{sin}(\omega t - \frac{\pi}{2}) \]

integrate \[ \int di = \frac{E_m}{L} \int \sin \omega t \, dt \]

\[ \text{Current amplitude} = \frac{E_m}{\omega L} = \frac{30}{2\pi \times 10^3} \times 5 \times 10^3 \]

a) \[ f = 10^3 \text{Hz} \]

\[ \text{Current Ampl.} = \frac{30}{2\pi \times \frac{3}{8} \times 5 \times 10^3} = \frac{30}{100} = 0.3 \times 10^{-1} \]

\[ = \frac{3}{10^{-1}} = 0.0955 \text{Amps} \]

b) \[ f = 8 \times 10^3 \]

\[ \text{Current ampl} = \frac{0.0955}{8} = 0.0119 \text{Amps} \]
Problem 31-43. RLC

\[ L = 88 \times 10^3 \, \text{H} \quad R = ? \quad C = 0.94 \times 10^{-6} \, \text{F} \quad f = 43 \, \text{kHz} \]

\[ \phi = 75^\circ \times \frac{\pi}{180^\circ} \quad R = ? \]

\[ \tan \phi = \frac{X_L}{X_C} = \frac{\omega L - \frac{1}{\omega C}}{R} \]

\[ R = \frac{2\pi fL - \frac{1}{\omega C}}{\tan \phi} = \frac{2\pi 930 \times 88 \times 10^3}{2\pi 930 \times 0.94 \times 10^{-6}} \]

\[ R = \frac{514.2 - 182.06}{3.732} = 89.2 \]
Problem 31-51

**RLC**

**Fig 31-33**

\[ R = 100 \, \Omega \]
\[ L_1 = 1.7 \times 10^{-3} \, H \]
\[ L_2 = 3 \times 10^{-3} \, H \]
\[ C_1 = 6 \times 10^{-6} \, F \]
\[ C_2 = 5 \times 10^{-6} \, F \]
\[ C_3 = 2 \times 10^{-6} \, F \]

(a) Inductors in series: \( L_{eq} = L_1 + L_2 \)

\( C_{eq} = \frac{1}{R \left( L_1 + L_2 \right)} \)

\[ C_{eq} = C_1 + C_2 + C_3 \]

(b) \( f \) does not change when \( R \) changes.

(c) \( f \) increase \( \rightarrow \) \( f \) decrease.

(d) \( C_3 \) removed \( \rightarrow \) \( C_{eq} \) decreases \( \rightarrow \) \( f \) increases.

\[ f = \frac{1}{2\pi \sqrt{L_{eq} C_{eq}}} = \frac{1}{2\pi \sqrt{(4 \times 10^{-3})(10^{-5})}} \]

\[ f = 796 \, Hz \] resonant frequency
Problem 31-63  AC Power

DC: \( I^2 \) power loss \( I^2R \)

\[ I_0 = 3.6\, \text{A} \] & \( k \) power loss \( P_k = \frac{I_0 V_0 \cos \theta}{2} \)

\[ d = 0 \] \( I^2R = \frac{I_0 V_0}{2} = \frac{I_0 I_0 R}{2} \)

\[ \therefore \quad I^2 = \frac{I_0^2}{2} \] \( I = \frac{I_0}{N} = \frac{2.6}{3} = 1.84\, \text{A} \)
Problem 31-59  Series RLC

Fig 31-7

\[ C \rightarrow \text{Loss is max} \]

\[ P_{\text{Loss}} = \frac{V_0^2}{2} \cos \phi = \frac{1}{2} I_0^2 R = \frac{V_0^2}{2} \frac{R}{Z_{RLC}^2} \]

since \( V_0 = I_0 Z_{RLC} \)

Thus occur at resonance \( \omega_d = \frac{1}{\sqrt{L C}} \)

\[ C = \frac{1}{\omega_d^2 L} = \frac{1}{(4 \pi^2 60)^2 \times 6 \times 10^{-2}} \]

\[ C = 1.67 \times 10^{-4} \text{ F} \]

b) \( P_{\text{Loss}} \) is min when \( Z_{RLC}(\omega) \) is max.

\[ Z_{RLC}(\omega) = R^2 + (X_L - X_C)^2 \]

\[ Z_{RLC}(\omega_d) = R^2 + (\omega L - \frac{1}{\omega C})^2 \]

when \( C \rightarrow 0 \)

c) max dissip. rate

d) phase angle

e) power factor

\[ \phi = \tan^{-1} 0 = 0 \]

\[ \cos \phi = 1 \]

f) min dissip. rate

g) phase angle

\[ Z_{RLC} \rightarrow \infty \]

\[ P_{\text{Loss}} = 0 \]

phase angle \( \phi = \tan^{-1} \left( \frac{-\infty}{R} \right) = -\frac{\pi}{2} \)

\[ \cos \phi = 0 \]
Problem 31-64: Transformer

\[ V_p = 100 \text{ V} \]
\[ N_p = 50 \quad N_s = 500 \]

\[ \frac{V_s}{V_p} = \frac{N_s}{N_p} \]

\[ V_s = V_p \cdot \frac{N_s}{N_p} = 100 \cdot \frac{500}{50} = 100 \text{ V} \]

\[ V_s = 100 \text{ V} \]
Problem 31 - 69 Series RLC

\[ E_{\text{MAX}} = 125 \text{ V} = V_0 \quad I_0 = 3.2 \text{ A} \]

\[ i \text{ leads } \phi \text{ by } 0.982 \text{ rad}. \]

\[ \phi = -0.982 \text{ rad}. \]

\[ a) \quad Z_{\text{RLC}} = \frac{V_0}{I_0} = \frac{125}{3.2} = 39.1 \Omega \]

\[ b) \quad R = \ ? \]

\[ \tan \phi = \frac{(X_L - X_C)}{R} \]

\[ Z_{\text{RLC}}^2 = R^2 + \left( X_L - X_C \right)^2 \]

\[ Z_{\text{RLC}}^2 = R^2 + \frac{R^2 + X_L^2}{1 + \tan^2 \phi} = R^2 \left( 1 + \tan^2 \phi \right) \]

\[ R = \frac{Z_{\text{RLC}}}{(1 + \tan^2 \phi)^{1/2}} = \frac{39.1}{(1 + \tan^2 0.982)^{1/2}} \]

\[ R = 21.7 \ \Omega \]

\[ c) \quad \text{i capacitive or inductive} \]

\[ X_L - X_C = R \tan(0.982) < 0 \]

\[ \therefore X_L < X_C \]

\[ \text{Capacitive. (also since i leads } \phi \text{)} \]

\[ "i \text{ cc}" \]
Problem 31-95 - LC circuit

\[ C = 7 \times 10^{-6} \text{ F} \quad V(0) - 12 \text{ Volts} = \frac{9(\omega)}{C} \]

\[ f = 715 \text{ Hz} \quad L = ? \]

\[ \omega = 2\pi f = \omega = \frac{1}{\sqrt{LC}} \]

\[ L = \frac{1}{C} \quad \frac{1}{\frac{4\pi^2 f^2}{L}} = \frac{1}{7 \times 10^{-6} \quad 4\pi^2 \quad (715)^2} \]

\[ L = 7.08 \times 10^{-3} \text{ H} \]

12 Volts was a Red Herring.