

MTG 3203  
Lecture from 9/15/09

**Goal:** Given line segments of length 1, length  $m$ , and length  $n$ , construct line segments of length:

- 1)  $m + n$  and  $m - n$  (or  $n - m$  if  $n > m$ )
- 2)  $mn$
- 3)  $m/n$
- 4)  $\sqrt{m}$

1) Constructing  $m + n$  and  $m - n$  (we will assume  $n < m$ ):

- a) Draw a line, and on it draw a segment  $\overline{AB}$  whose length is  $m$ .
- b) Draw a circle centered at point  $B$  whose radius is  $n$ .
- c) Mark the intersection point between  $A$  and  $B$  as  $C$ , mark the other intersection as  $D$ .
- d) The segment  $\overline{AC}$  has length  $m - n$  and the segment  $\overline{AD}$  has length  $m + n$ .

2) Constructing  $mn$ :

- a) Draw a triangle  $ABC$  such that  $\overline{AB}$  has length  $m$  and  $\overline{AC}$  has length 1.
- b) Extend the line  $\overleftrightarrow{AC}$ , and draw the segment  $\overline{AD}$  on  $\overleftrightarrow{AC}$  such that the length of  $\overline{AD}$  is  $n$  and the direction of  $\overleftrightarrow{AC}$  is the same as  $\overleftrightarrow{AD}$ .
- c) Draw the line parallel to  $\overline{BC}$  passing through  $D$ . This line will intersect  $\overleftrightarrow{AB}$  at a point. Call it  $E$ .
- d) The segment  $\overline{AE}$  has length  $mn$ .

3) Constructing  $m/n$ :

- a) Draw a triangle  $ABC$  such that  $\overline{AB}$  has length  $m$  and  $\overline{AC}$  has length  $n$ .
- b) Extend the line  $\overleftrightarrow{AC}$ , and draw the segment  $\overline{AD}$  on  $\overleftrightarrow{AC}$  such that the length of  $\overline{AD}$  is 1 and the direction of  $\overleftrightarrow{AC}$  is the same as  $\overleftrightarrow{AD}$ .
- c) Draw the line parallel to  $\overline{BC}$  passing through  $D$ . This line will intersect  $\overleftrightarrow{AB}$  at a point. Call it  $E$ .
- d) The segment  $\overline{AE}$  has length  $m/n$ .

4) Constructing  $\sqrt{m}$ :

- a) Make a segment of length  $(m + 1)/4$ .
- b) Draw a circle of radius  $(m + 1)/4$ . Mark a diameter on the circle, and call its endpoints  $A$  and  $B$ .
- c) Make a segment of length  $(m - 1)/2$  (or  $(1 - m)/2$  if  $m < 1$ ).
- d) Draw a circle centered at  $B$  with radius  $(m - 1)/2$ .
- e) The two circles will intersect at two points. Call one of these points  $C$ .
- f) The segment  $\overline{AC}$  has length  $\sqrt{m}$ .