Dominance

Given two functions $f(n)$ and $g(n)$, we say $f(n)$ *dominates* $g(n)$ if

$$\lim_{n\to\infty} \frac{f(n)}{g(n)} = \infty \qquad \text{or} \qquad \lim_{n\to\infty} \frac{g(n)}{f(n)} = 0$$

The two conditions are equivalent (if the first is infinite, the second is zero, and vice versa). The official mathematical term for "dominates" is "asymptotically greater". I think "dominates" sounds cooler. Two functions are of equal dominance if $\lim_{n\to\infty} f(n)/g(n) = L$ for some number $L \neq 0$, $L \neq \infty$. Using l'Hopital's Rule and other means, we can rank common functions in terms of dominance:

Function	Thoughts
Bounded functions and constants	Includes $(-1)^n$, $\cos(n)$, $\arctan(n)$
Logs	$\ln(n)$
Polynomials	If $a > b$, then n^a is more dominant than n^b.
Exponentials	If $a > b$, then a^n is more dominant than b^n.
Factorials	$n!$

In that table, the functions further down are more dominant. The final useful rule for dominance is:

If $f(n)$ is more dominant than $g(n)$, then $f(n)$ has the same level of dominance as $f(n)+g(n)$.

Dr. Dan's Dominance Test: If $h(x)$ is the most dominant term in $f(x)$ and $j(x)$ is the most dominant term in $g(x)$, then the two series

$$\sum_{n=1}^{\infty} \frac{f(x)}{g(x)} \qquad\qquad \sum_{n=1}^{\infty} \frac{h(x)}{j(x)}$$

both converge or both diverge.