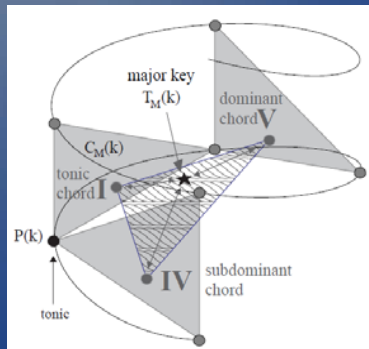


Modeling Tonality: Applications to Music Cognition



Introduction

- Music cognition is a complex task requiring the integration of information at many different levels.
- The mind is so adept at organizing and extracting meaningful patterns when listening to music that we are often not even aware of it.
- What is it we know that causes us to hear one pitch as being more stable than others?

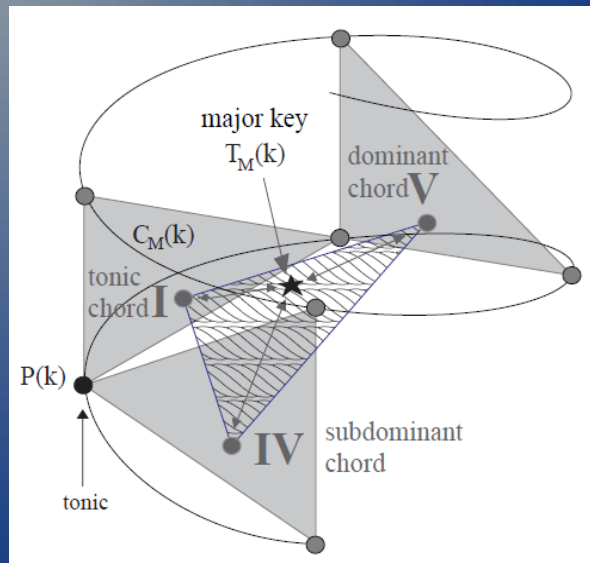
Introduction

- This paper proposes a mathematical model for tonality.
- The model uses spatial proximity to represent perceived distances between musical entities.
- This model is used to design a computational algorithm to mimic human decisions in determining keys.

The Spiral Array Model

- The Spiral Array is a hierarchical model, which generates representations for pitches, intervals, chords and keys within a single spatial framework.
- This allows for comparisons among elements from different hierarchical levels.
- Which allows for the representation of higher level tonal elements as aggregates of their lower level components.

The Spiral Array Model



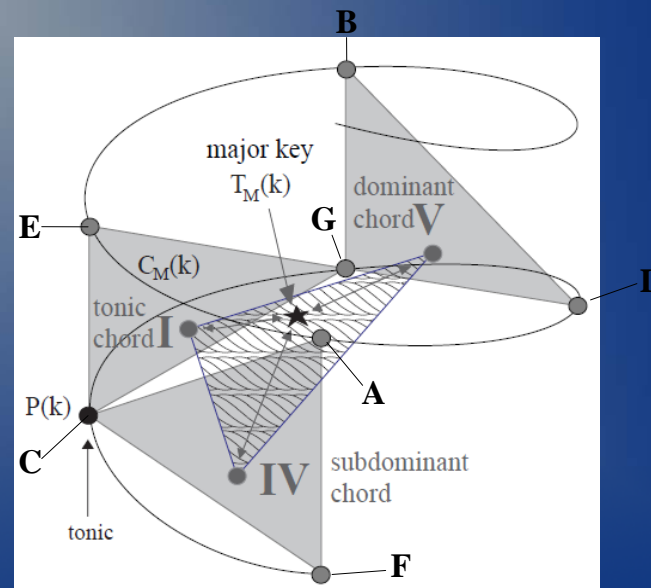
The Spiral Array Model

- The Spiral Array model derives from a three-dimensional realization of the Harmonic Network.
- The representations for intervals, chords and keys are constructed as mathematical aggregates of spatial representations of their component parts.
- Distances between tonal entities as represented spatially in the model correspond to perceived distances among sounding entities.

The Spiral Array Model

- Adjacent pitches are related by intervals of perfect fifths ascending up the spiral.
- Adjacent pitches are related by intervals of perfect fourths descending down the spiral.
- Pitches vertically above one another are related by major thirds.
- Pitches are indexed by their number of perfect fifths from C.

The Spiral Array Model



The Spiral Array Model Algorithm

- The pitch C is arbitrarily set at the point $[0,1,0]$.
- $P(k)$ denotes the point on the spiral representing a pitch of index k .

$$P(k+1) \stackrel{\text{def}}{=} \mathbf{R} \cdot P(k) + \mathbf{h},$$

where $\mathbf{R} = \begin{bmatrix} 0 & 1 & 0 \\ -1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}$ and $\mathbf{h} = \begin{bmatrix} 0 \\ 0 \\ h \end{bmatrix}$

The Spiral Array Model Algorithm

- Positions representing pitches four indices, or a major third, apart are related by a simple vertical translation.

$$P(k+4) = P(k) + 4 \cdot \mathbf{h}.$$

The Spiral Array Model Algorithms

- A chord is represented by a weighted average of its component pitch positions: the root $P(k)$, the fifth $P(k+1)$, and the third $P(k+4)$ for major triads.
- The weights w_i on the pitch positions represent the importance of the pitch to the generated chord.

$$C_M(k) \stackrel{\text{def}}{=} w_1 \cdot P(k) + w_2 \cdot P(k+1) + w_3 \cdot P(k+4),$$

where $w_1 \geq w_2 \geq w_3 > 0$ and $\sum_{i=1}^3 w_i = 1$.

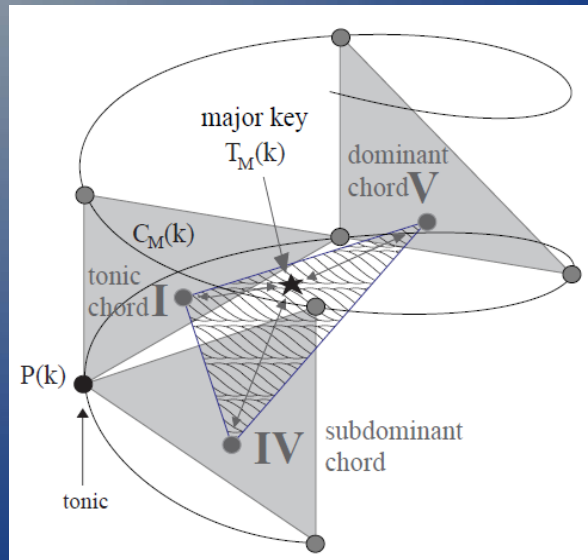
The Spiral Array Model Algorithms

- The representation for a major key $T_M(k)$ is the weighted average of its tonic triad ($C_M(k)$), dominant triad ($C_M(k+1)$) and subdominant triad ($C_M(k-1)$) representations.
- The weights correspond to each chord's significance in the key.

$$T_M(k) \stackrel{\text{def}}{=} \omega_1 \cdot C_M(k) + \omega_2 \cdot C_M(k+1) + \omega_3 \cdot C_M(k-1),$$

where $\omega_1 \geq \omega_2 \geq \omega_3 > 0$ and $\sum_{i=1}^3 \omega_i = 1$.

The Spiral Array Model



Properties of the Spiral Array Model

1. Perceptually close intervals should be represented by shorter inter-pitch distances.
2. Each chord representation is closest to its root, followed by the fifth, then the third.
3. The average position of two pitches an interval of a half step apart should be closest to the key related to the upper pitch; and, the average position of two pitches an interval of a perfect fourth apart should be closest to the key related to the upper pitch.

Finding the Key of a Melody

- In the Spiral Array model, musical information is condensed, or aggregated, and represented by a single point.
- The model is used to create a key-finding algorithm called the Center of Effect Generator (CEG).
- The CEG algorithm uses the model to identify keys through finding a distance-minimizing representation.

Center of Effect Generator Algorithm

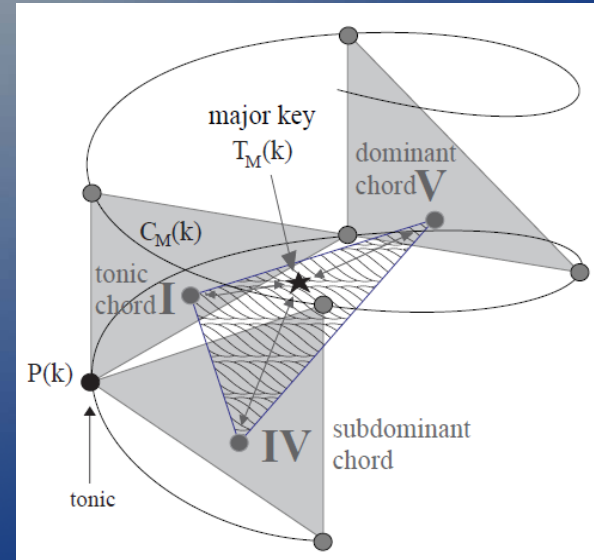
- As pitches in a melody are sounded, the geometric shape defined by the pitch positions becomes increasingly more complex.
- The algorithm collapses the pitch information down to a single point, the center of effect (c.e.).
- The pitches combine to create an object in space, a point which is the composite sum of the pitch positions.

Center of Effect Generator Algorithm

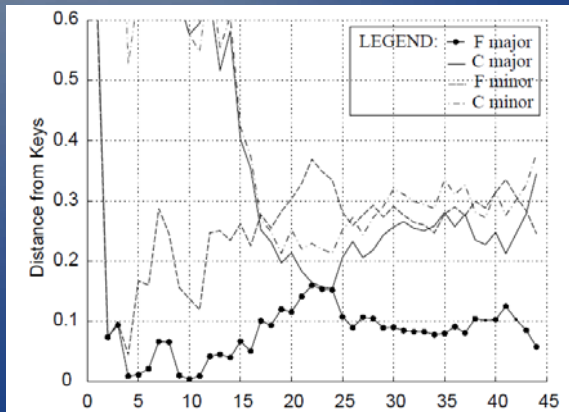
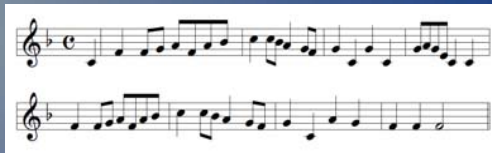
- Since keys are defined as points in space, one can compute the distance between the c.e. and nearby keys, to determine which key is closest to the c.e.
- If the i -th note is represented in the Spiral Array by pitch position p_i and has duration d_i , then the aggregate center at the i -th pitch event is defined as:

$$c_i \stackrel{\text{def}}{=} \sum_{j=1}^i d_j \cdot p_j.$$

The Spiral Array Model



Example: “Simple Gifts”



Comparison to other Key-Finding Models

- The model was compared to Longuet-Higgins & Steedman’s Shape-Matching Algorithm and to Krumhansl & Schmuckler’s Probe Tone Profile Method.
- All three models were applied to the fugue subjects in Book I of Bach’s Well Tempered Clavier.
- The CEG required 3.75 pitch events, the PTPM 5.25, and the SMA 8.71 to determine the correct key.
- The CEG performance is close to optimal, as a minimum of three pitches are needed to accurately determine a key.

Conclusions

- The approach used in this research is computational, and mimics human music cognitive abilities.
- The algorithm used performs well and should be considered as a method of modeling human cognition in music.
- The model can serve as a research tool for testing hypotheses about human cognition in music.
- Future extensions of the model might incorporate both forward and retroactive decision-making.