1. Compute the following by using

\[ A = \begin{bmatrix} 2 & -1 & 5 \\ 3 & 4 & 1 \end{bmatrix}, \quad B = \begin{bmatrix} 1 & 0 & -2 \\ 2 & 3 & 4 \end{bmatrix}, \quad \text{and} \quad v = \begin{bmatrix} 1 \\ -2 \\ 5 \end{bmatrix}. \]

a) \( A + 2B \) and \( (A + 2B)^T \)

b) \( (A + 2B)v \) and \( ((A + 2B)v)^T \)

c) \( v^T(A + 2B)^T \)
2. Let $A$ be a $3 \times 3$ matrix such that the $ij$-entry $a_{ij}$ of $A$ is given by $ij$. Write the matrix $A$. Determine if $A$ is diagonal, symmetric, skew-symmetric, or none of these.

3. Obtain a vector $v$ by rotating the vector $u = \begin{bmatrix} 2 \\ -1 \end{bmatrix}$ by $135^\circ$ in the counter-clockwise direction.
4. Prove that if $A$ is an $n \times n$ square matrix, then $A + A^T$ is a symmetric matrix.

5. Consider the following system of linear equations:

\[
\begin{align*}
    x_1 - x_3 - 2x_4 - 8x_5 &= -3 \\
    -2x_1 + x_3 + 2x_4 + 9x_5 &= 5 \\
    3x_1 - 2x_3 - 3x_4 - 15x_5 &= -9.
\end{align*}
\]

a) Write the system in the matrix-vector form by finding the coefficient matrix $A$, variable vector $x$, and the right-hand-side vector $b$. Also form the augmented matrix for the system.
b) Use Gaussian elimination to transform the augmented matrix you obtained in part (a) to its reduced row echelon form. Describe each operation performed on the matrix.

c) Determine if the system is consistent or inconsistent. If consistent, determine the basic and free variables of the system and find the general solution of the system.