PART I - Basic Skills

Each question is worth 3 points. Write your answer in the space provided after each question. No partial credit is awarded.

1. Evaluate the following for \( m = 3 \) and \( n = -2 \):

\[
\frac{|3|-1-2|}{|3|(-2)|} = \frac{3-2}{151} = \frac{1}{5}
\]

Answer: \( \frac{1}{5} \)

2. Factor: \((2a)^3 + b^3\)

\[
= (2a+3)((2a)^2 - 2ab + b^2)
= (2a+3)(4a^2 - 2ab + b^2)
\]

Answer: \((2a+b)(4a^2-2ab+b^2)\)

3. Simplify and write with positive exponents: \(\left(\frac{3x^{-1}}{4y^{-1}}\right)^{-2}\)

\[
= \frac{3^2}{4^2} \times \frac{2}{y^2} = \frac{16x^2}{9y^2}
\]

Answer: \(\frac{16x^2}{9y^2}\)
4. Write 0.0514 in scientific notation.

\[
5.14 \times 10^{-2}
\]

Answer: \(5.14 \times 10^{-2}\)

5. Factor by grouping: \(3x^2 + 6x - x - 2\)

\[
= 3x(x + 2) - (x + 2)
= (3x - 1)(x + 2)
\]

Answer: \((3x - 1)(x + 2)\)

6. Rationalize the denominator: \(\frac{1}{\sqrt{2} - 1}\)

\[
= \frac{\sqrt{2} + 1}{2 - 1} = \sqrt{2} + 1
\]

Answer: \(\sqrt{2} + 1\)

7. Write in the standard form \(a + bi\): \(\frac{2 + 3i}{1 - i}\)

\[
= \frac{2 + 2i + 3i + 3i^2}{1 - i^2}
= \frac{2 + 5i - 3}{1 + 1}
= \frac{-1 + 5i}{2}
\]

Answer: \(-\frac{1}{2} + \frac{5}{2}i\)

8. Solve and express your answer using interval notation: \(-3(x - 1) < 12\)

\[
x - 1 > -4
\]

\[
x > -3
\]

Answer: \((-3, \infty)\)
PART II - Problem Solving skills

Each problem is worth 10 points. You must show your work to get full credit. Answers without the relevant steps will not get credit even if the answer is correct.

1. Perform the operation and simplify:

\[
\frac{x}{(x-1)^2} + \frac{2}{x} - \frac{x+1}{x^3-x^2} = \frac{x}{(x-1)^2} + \frac{2}{x} - \frac{x+1}{x(x+1)} - \frac{1}{x-1}
\]

\[
= \frac{x^2 + 2x(x^2-2x+1) - (x+1)(x-1)}{x^2(x-1)^2}
\]

\[
= \frac{x^2 + 2x^3 - 4x^2 + 2x - x + 1}{x^2(x-1)^2}
\]

\[
= \frac{3x^3 - 5x^2 + 2x + 1}{x^2(x-1)^2}
\]

2. Solve for \(x\):

\[
(x-2)(x+2) \left( \frac{x}{x+2} - \frac{1}{x-2} \right) = \frac{4}{x-2} (x-1)(x+2)
\]

\[
\text{LCM} = (x-2)(x+2)
\]

\[
\Rightarrow x(x-2) - (x-4) = 4(x+2)
\]

\[
x^2 - 2x - x + 4 = 4x + 8
\]

\[
x^2 - 6x - 4 = 0
\]

\[
x = \frac{6 \pm \sqrt{36 + 16}}{2} = \frac{6 \pm 10}{2}
\]

\[
x = \frac{16}{2} = 8
\]

\[
x = \frac{-4}{2} = -2
\]

\[
x = \frac{8}{2} = 4
\]

\[
x = \frac{-2}{2} = -1
\]
3. Solve for $x$ in the complex number system:

$$x^4 - 5x^2 + 4 = 0$$

Let $u = x^2$ then, $u^2 - 5u + 4 = 0$

$$(u-1)(u-4) = 0 \implies u = 1 \text{ or } u = 4$$

If $u = 1$ then $x^2 = 1 \implies x = \pm 1$

If $u = 4$ then $x^2 = 4 \implies x = \pm 2$

Therefore, $\{x = \pm 1, \pm 2\}$

4. Consider the following equation: $x^2 - 6x + 13 = 0$

a) Find the discriminant.

$$b^2 - 4ac = (-6)^2 - 4(1)(13) = 36 - 52 = -16$$

b) Determine the number and type of solutions.

$$b^2 - 4ac = -16 < 0$$

Two distinct nonreal complex solutions.

c) Find the solution(s).

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{-(-6) \pm \sqrt{-16}}{2}$$

$$= \frac{6 \pm 4i}{2} = 3 \pm 2i$$

Therefore, $x = 3 \pm 2i$
5. Solve for \( x \):

\[
3 + \sqrt{21 - 2x} = x
\]

\[
(\sqrt{21 - 2x})^2 = (x-3)^2
\]

\[21 - 2x = x^2 - 6x + 9\]

\[-x^2 - 4x - 12 = 0\]

\[(x - 6)(x + 2) = 0\]

\[x - 6 = 0 \text{ or } x + 2 = 0\]

\[x = 6 \text{ or } x = -2\]

\[\text{Check}\]

\[x = 6 \Rightarrow 3 + \sqrt{21 - 12} = 6\]

\[3 + 3 = 6 \checkmark\]

\[x = -2 \Rightarrow 3 + \sqrt{75} \neq 6\]

\[3 + 5 \neq 6 \times\]

\[\text{Only solution is } x = 6\]

6. For \( z = i \) and \( w = 2 + 3i \), write the following in standard form:

a) \( \bar{z} , \bar{w} , z^{15} \)

\[\bar{z} = -i\]

\[\bar{w} = 2 - 3i\]

\[2^{15} = 2^3 = 1 \cdot 2 = -i\]

b) \( w\bar{w} \)

\[= (2+3i)(2-3i) = 4 + 9 = 13\]

c) \( \frac{z-\bar{w}}{} \)

\[= \frac{i - (2-3i)}{} = \frac{1 - 2 + 3i}{-2 + 4i} = \frac{-2 + 4i}{-2 - 4i} = -2 - 4i\]
7. Solve and express your answer using interval notation:

\[|1 - 4x| - 7 \leq -2\]

\[\frac{3}{2} \geq x > -1 \implies -1 \leq x \leq \frac{3}{2} \implies [-1, \frac{3}{2}]\]

8. How much water must be evaporated from 240 gallons of a 3% salt solution to produce a 5% salt solution?

<table>
<thead>
<tr>
<th></th>
<th>%</th>
<th>Amount</th>
<th>Amount of pure salt</th>
</tr>
</thead>
<tbody>
<tr>
<td>Salt soln.</td>
<td>3%</td>
<td>240</td>
<td>0.03 \times 240</td>
</tr>
<tr>
<td>Water</td>
<td>0%</td>
<td>x</td>
<td>0</td>
</tr>
<tr>
<td>Final soln</td>
<td>5%</td>
<td>240 - x</td>
<td>0.05 \times (240 - x)</td>
</tr>
</tbody>
</table>

So,

\[0.03 \times 240 = 0.05 \times (240 - x)\] — mult. by 100.

\[3 \times 240 = 5 \times (240 - x)\]

\[720 = 1200 - 5x\]

\[5x = 480 \implies x = 96\] gallons of water must be evaporated.