Sunshine State Scholars
Regional Competition - Solutions
January 8, 2004

Problem 1:

The liquid propellant fueling of the external tank on the space shuttle commences six hours prior to launch, and is a continuous process. The external tank is a cylinder 153 ft. tall and 27 ft. 6 in. in diameter. During the fueling of the liquid propellant ice accumulates uniformly on the lower two thirds of the outside of the tank at a rate of 3/8 inch per hour; no ice forms on the bottom. At the time of launch, how much does the accumulated ice add to the overall weight of the tank? Assume that ice weighs 56.25 lbs per cubic foot.

Solution:

The height of the portion of the tank that accumulates ice is 153 x $\mathbf{\frac{2}{3}}$ = 102 feet. The "thickness" of the ice when the shuttle is launched is $\mathbf{\frac{3}{8}}$ x 6 = 2.25 inches = .1875 feet. The diameter of the cylinder that is the portion of the tank that is iced is 27.5 feet. The diameter of the same cylinder with the ice is 27.875 feet. The volume of the cylinder without the ice is $V_1 = \pi \times \left(\frac{27.5}{2}\right)^2 \times 102 \approx 60{,}583.65$ cubic feet. The volume of the cylinder with the ice is $V_2 = \pi \times \left(\frac{27.875}{2}\right)^2 \times 102 \approx 62{,}247.2$ cubic feet. The volume of ice on the tank is $V_2 - V_1 \approx 62{,}247.2 - 60{,}583.65 = 1{,}663.55$ cubic feet. The weight of that ice is $W \approx 1{,}663.55 \times 56.25 \approx 93{,}575$ lbs.

Problem 2:

Hydrazine, N_2H_4, is a toxic liquid derived from ammonia. It is very explosive in the presence of air and other oxidizing agents. One important use of hydrazine is in the steering rockets used on spacecraft. Liquid hydrazine reacts with liquid dinitrogen tetroxide, N_2O_4, to produce nitrogen and water gases. The controlled and directional release of these gases produces a steering effect on a spacecraft.

a. If the reaction were carried out on earth, what volume of gas would be produced by the reaction of 10.0 kg of hydrazine in an excess of dinitrogen tetroxide if it were contained at 1.0 atm of pressure and a temperature of –40.0°C?

b. Calculate the mass of the dinitrogen tetroxide required to completely react with 10.0 kg of hydrazine?

Solution:

a. A balanced equation for the reaction between hydrazine and dinitrogen tetroxide is $\mathbf{2N_2H_4 + N_2O_4 \rightarrow 3N_2 + 4H_2O}$. To produce 7 moles of gas consisting of 3 moles of nitrogen and 4 moles of water vapor will require 2 moles of hydrazine. The number of moles of gas produced from 10.0 kg of hydrazine can be calculated as follows. (The atomic number of nitrogen is 14.01 and the atomic number of hydrogen is 1.01. The atomic weight of hydrazine is 32.06.)

$$\mathbf{10\,kg \times \frac{1000\,gm}{kg} \times \frac{1\,moles\ N_2H_4}{32.06\,gm} \times \frac{7\,moles\ gas}{2\,moles\ N_2H_4} \approx 1092\,moles\ of\ gas}$$

The volume of a given number of moles of a gas at a given temperature and pressure is given by $V = \frac{nRT}{P}$. Negative 40 degrees Centigrade is 233.15 $\mathbf{K^{\circ}}$.

$$V = \frac{\mathbf{1092\,moles \times .0821\frac{L\,atm}{mole\,K^{\circ}} \times 233.15\,K^{\circ}}}{\mathbf{1\,atm}} \approx \mathbf{2.09 \times 10^4\ Liters}$$

b. The atomic weight of dinitrogen tetroxiide is 92.01. The number of kilograms of dinotrogen tetroxide required to completely react with 10.0 kilograms of hydrazine can be calculated as follows.

$$\mathbf{10.0\,kg\,N_2H_4 \times \frac{1\,kmole\,N_2H_4}{32.06\,kg\,N_2H_4} \times \frac{1\,kmole\,N_2O_4}{2\,kmole\,N_2H_4} \times \frac{92\,kg\,N_2O_4}{1\,kmole\,N_2O_4} \approx 14.3\,kg\,N_2O_4}$$

Problem 3:

Susan and Jim are both dark haired, Susan has blue eyes and Jim has brown eyes. Terri, their first child, has decided that since she has blonde hair and blue eyes, she could not be Susan and Jim's daughter (all of her siblings are brown haired and blue eyed). Could Terri be Susan and Jim's daughter? Is it possible that she is not their daughter? Use diagrams and probability to explain your answer. (Note that "blonde hair" and "blue eyes" are recessive genes.)

Solution:

One set of Terri's chromosomes comes from her mother and the other from her father. The traits of hair and eye color are non-sex linked and can be displayed as a monohybrid cross for each characteristic. Blonde hair and blue eyes are recessive genes. If both her parents carried these recessive genes, then there is the possibility that Terri would have both blonde hair and blue eyes even though her parents displayed the dominant characteristics. Let B represent a dominant allele for hair color and b represent a recessive allele for blonde hair. Similarly let E represent a dominant allele for eye color and let e represent a recessive allele for blue eyes. Susan could be "BB" or "Bb" for hair color but must be "ee" for eye color. Jim could be "BB" or "Bb" for hair color and he could be either "EE" or "Ee" for eye color. If a child of Susan and Jim has blonde hair and blue eyes both parents must carry the recessive genes. Susan must be "Bbee" and Jim must be "BbEe". It is possible, of course, that Susan and Jim do not carry this combination of alleles and Terri is not their daughter. Suppose Susan is "Bbee" and Jim is "BbEe".

The Punnet square for hair color is as follows.

	B	b
B	BB	Bb
b	bB	bb

The probability that a child has both recessive genes and blonde hair is _.

The Punnet square for eye color is as follows.

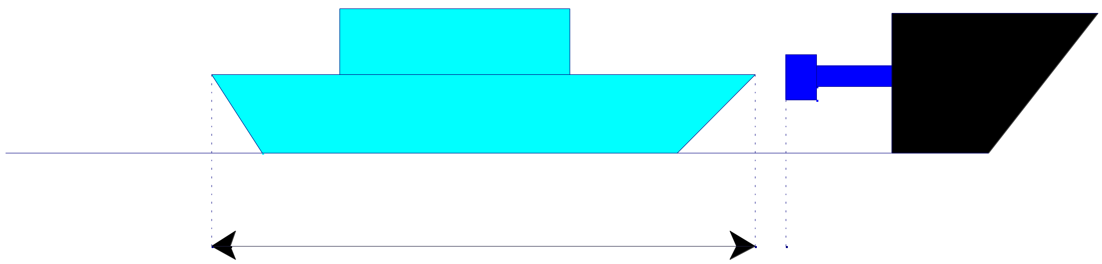
	e	e
E	Ee	Ee
e	ee	ee

The probability that a child has both recessive genes and blue eyes is _.

Since these characteristics are independent, the probability that a child will have both blonde hair and blue eyes is 1/8. In this case there is a finite probability that a child of Susan and Jim will have both blonde hair and blue eyes and Terri could be their daughter.

Problem 4:

A ship that is 300 meters long pulls into a docking slip. The front of the ship stops 1 cm short of the docking clamps. The engineer, who has a mass of 100kg, is standing at the front of the ship. He runs to the other end of the ship and the docking clamps just catch. Assuming the center of mass of the ship and contents is at the center of ship (except for the engineer) find the mass of the ship.



Solution:

The ship and the engineer form a closed system whose momentum must remain constant at zero. The momentum of the engineer running from the front of the ship to the back must equal the momentum of the ship moving one centimeter in the opposite direction. The engineer will run a total distance of 300 - .01 meters in time ***T***. In the same time the ship will move .01 meters in the opposite direction. Let ***M*** be the mass of the ship.

$$\frac{.01M}{T} = \frac{100(300 - .01)}{T}$$

$$M = \frac{29{,}999}{.01} = 2.9999 \times 10^6\,kg$$

A second method of solution involves the analysis of the "center of mass" of the closed system consisting of the ship and the engineer. The center of mass of the system must remain at a fixed position in space. Set up a coordinate system with zero at the bow of the ship prior to docking and the positive direction toward the back of the ship. Let x_i be the initial ***x*** coordinate of the center of mass in this system and let x_f be the final ***x*** coordinate of the center of mass in the same system.

$$x_i = \frac{(M \times 150) + (100 \times 0)}{M + 100}$$

$$x_f = \frac{M \times (150 - .01) + 100 \times (300 - .01)}{M + 100}$$

$$x_i = x_f$$

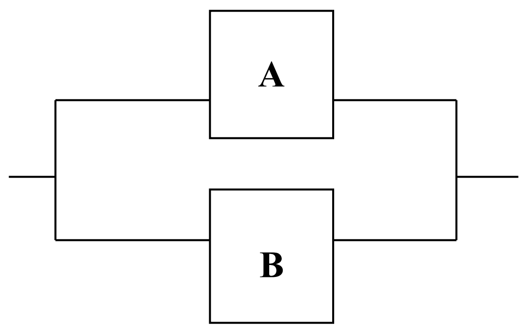
$$150M = 150M - .01M + 29999$$

$$M = \frac{29999}{.01} = 2.9999 \times 10^6\ kg$$

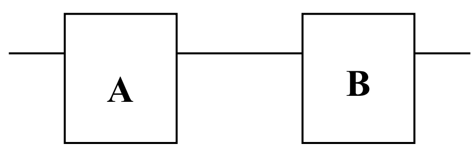
Problem 5:

One of the engineering strategies when building complex systems, such as a space shuttle, is the concept of redundancy. A system consists of many components, all of which are subject to failure. In order to minimize the system failure rate, components are installed in such a way that if one fails there are others built in to perform the function of the failed component. In electrical systems (and others as well), components can be arranged in parallel or in series. If two components are in parallel, the system will work as long as one of the components works. If two components are in series, the system will work only if both components work. Schematically this is shown in the following diagrams.

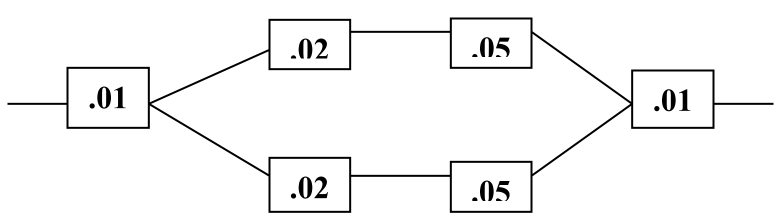
Components A and B in parallel:



Components A and B in series:

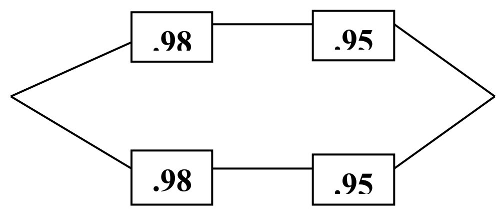


Assume that all the components diagrammed in the following system operate independently. The probability that any individual component will fail is listed in the box representing that component. What is the probability that this system will work properly the first time that it is used?



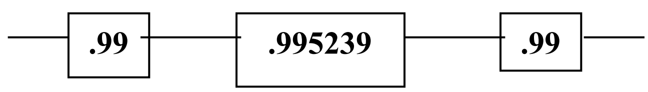
Solution:

Consider the following portion of the system that is in parallel. The probability of each component working correctly is given.



The probability that the top two components both work correctly is (.98)(.95). The probability that the bottom two components work correctly is also (.98)(.95). The probability that either the top or the bottom works correctly is the probability of the top working plus the probability of the bottom working minus the probability that both will work. (In summing the probability of the top and bottom working the probability that both will work has been counted twice.) Therefore the probability that this portion of the system will work correctly is $(.98)(.95) + (.98)(.95) - (.98)(.95)(.98)(.95) = 2(.98)(.95) - (.98)^2(.95)^2 = .995239$.

Now the entire system can be viewed as a series as shown in the following diagram. The probabilities of each component working correctly are given.



The probability that the entire system will work correctly is:

$$(.99)(.995239)(.99) \approx .975$$

Problem 6:

The atom density of hydrogen in intergalactic space has been estimated to be about 1 H atom/cm^3. It is difficult to define a temperature at this very low density, but assume that it is about 1000 K.

a. What is the pressure of atomic hydrogen in intergalactic space?
b. What volume of space (in km^3) contains 1.0 g of H atoms?
c. The average distance a gas phase atom or molecule travels before colliding with another atom or molecule is called the mean free path. This quantity is inversely proportional to the gas pressure. If the mean free path of hydrogen atoms in a container with hydrogen at atmospheric pressure is about 10.0 nanometers, estimate the mean free path of a hydrogen atom in intergalactic space.

Solution:

a. The "Ideal Gas Law" is $\boldsymbol{PV = nRT}$. Solving for $\boldsymbol{P}$ produces $\boldsymbol{P = \frac{nRT}{V}}$. In this equation $\boldsymbol{n}$ is the number of atoms in a given volume divided by the number of atoms in a mole. In this case $\boldsymbol{n = 1/6.023 \times 10^{23}}$ moles. The gas constant is $\boldsymbol{R = 82.1 \frac{atm \times cm^3}{mole \times K^\circ}}$. Now the pressure can be calculated.

$$\boldsymbol{P = \frac{\frac{1}{6.023 \times 10^{23}} moles \times 82.1 \frac{atm \times cm^3}{moles \times K^\circ} \times 1000 K^\circ}{1 cm^3} = 1.363 \times 10^{-19} atm}$$

b. One gram of hydrogen atoms is one mole which is $\boldsymbol{6.023 \times 10^{23}}$ atoms. Since the density is one atom in one cubic centimeter, this is

$$\boldsymbol{6.023 \times 10^{23} cm^3 = 6.023 \times 10^8 km^3}.$$

c. The product of the "mean free path," λ, and the pressure, $\boldsymbol{P}$, will be constant. Let λ_{space} be the mean free path of hydrogen in space.

$$\boldsymbol{\lambda_{space} \times 1.363 \times 10^{-19} atm = 10 nm \times 1 atm}$$

$$\boldsymbol{\lambda_{space} = \frac{10 nm \times atm}{1.363 \times 10^{-19} atm} \approx 7.34 \times 10^{19} nm = 7.34 \times 10^7 km}$$

Problem 7:

To transport the shuttle and the mobile launch assembly to the launch pad, a crawler is used which must lift the entire mass of 2,042,000 kilograms with four individual hydraulic jack systems. Each jack system consists of a 305 cm diameter main lifting piston and a pressure piston with a diameter of 10.2 cm. How much force must be applied to each of the smaller pistons to lift the weight?

Solution:

Each piston must lift $\frac{2{,}042{,}000}{4} = 510{,}500\,kg$. The area of the pressure piston is $A_{pressure} = \pi\left(\frac{10.2}{2}\right)^2 \approx 81.71\,cm^2 = .008171\,m^2$. The area of the main lifting piston is $A_{main} = \pi\left(\frac{305}{2}\right)^2 \approx 73{,}062\,cm^2 \approx 7.3062\,m^2$. The pressure on each piston is the force divided by the area. Let $F_{pressure}$ and $P_{pressure}$ be the force and pressure on the pressure piston. Similarly F_{main} and P_{main} are the force and pressure on the main piston. Since fluids are essentially incompressible, the pressure on each piston must be the same in order to hold the shuttle at a given level.

$$P_{pressure} = \frac{F_{pressure}}{.008171} = P_{main} = \frac{510{,}500 \times 9.8}{7.3062} \approx 684{,}747\,\frac{Newtons}{m^2}$$

$$F_{pressure} \approx .008171 \times 684{,}747 \approx 5595\,Newtons$$

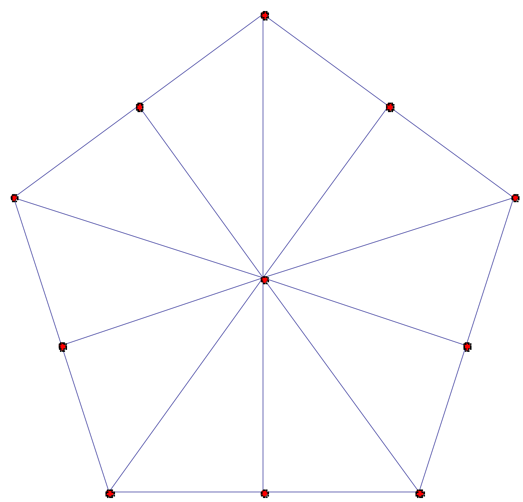
In order to lift the shuttle the force applied to each of the smaller pistons must exceed 5595 Newtons.

Problem 8:

A space station has the shape of a pentagonal prism with a cross section in the shape of a regular pentagon; each side of the pentagon is 20 meters long and the length of the station (perpendicular to the pentagons) is 30 meters. Debris from a distant exploding star passes through the region of space occupied by the station in a direction perpendicular to the pentagonal cross sections. The debris consists of objects traveling at 20 km/sec with an average density of 2 objects per cubic km. Determine the average number of objects that would impact the space station in an hour.

Solution:

Let ***N(t)*** be the average number of objects that will strike the space station in a time interval of length ***t***. Then ***N(t)*** will be the cross sectional area of the station that is perpendicular to the path of the objects times the density of the objects times their speed times ***t***. Let ***A*** be the area of the pentagonal cross section that is perpendicular to the object's path. Partition the pentagon into 10 right triangles as shown in the following figure.



The area of each triangle is $A_{triangle} = \frac{1}{2} \times 10 \times 10 \tan\left(54^{\circ}\right) \approx 68.8\, m^2$. The area of the pentagon is $A = 10 \times A_{triagle} \approx 10 \times 68.8 = 688\, m^2 = 6.88 \times 10^{-4}\ km^2$.

$$N(t) \approx 6.88 \times 10^{-4} \times 2 \times 20 \times 3600 \approx 99\ objects / hour$$

Problem 9:

The European green shore crab (*Carcinus maenas)*is an example of an invasive species introduced to U.S. in the ballast of ocean-going vessels. Although similar in size and appearance to the benign native green (or "yellow") shore crab (*Hemigrapsus oregonensis)*, the European variety is dramatically different in regard to diet and behavior. Not content to feed on algae and carrion like the West Coast native, *C. maenas* fuels a voracious appetite, consuming local clams and native crabs. One adult crab reportedly consumes 40 native clams in a day and is capable of devouring crabs equal to its own size.

a. One method is given in this example for the way in which an invasive organism can be introduced into a new habitat. Identify and discuss other methods by which organisms could be dispersed into new habitats. Consider both natural as well as non-natural modes.

b. List and discuss two effects (other than given by the example above) on an ecosystem that could occur if a new organism were introduced into the ecosystem?

Solution: