Sunshine State Scholars
Regional Competition - Solutions
January 17, 2001

#1. The introduction of antibiotics during the 1930s and their subsequent extensive use resulted in saving millions of lives that would have been lost without these remarkable substances. Unfortunately the widespread use, and in many cases misuse, of antibiotics also led to the development of bacterial strains that were highly resistant to various antibiotics. Development of resistant strains of bacteria has led to constant efforts by drug companies to identify and develop new antibiotics to combat bacterial infections. This experience has clearly shown that bacteria have an impressive ability to develop antibiotic resistant strains. With these issues in mind describe why you should take all of the antibiotics prescribed by your physician and not stop taking them when you begin to feel better.

Solution:

Failure to complete the dose of antibiotics prescribed by the physician might increase the probability of survival of a small number of the bacterial population that has managed to overcome the antibiotics and developed a resistance to its effects. The surviving population then can cause infections partially or totally resistant to the antibiotic and over time may render the antibiotic completely ineffective.

#2. People often use the term "blood sugar." This term refers to the compound, glucose, which is soluble in the blood. Glucose reacts with oxygen in several steps to yield CO_2 and H_2O.

$$C_6H_{12}O_6 + 6\,O_2\ (g) \rightarrow 6\,CO_2\ (g) + 6\,H_2O\ (l) \qquad \triangle H^\circ = -2816\ \frac{kJ}{mole}$$

How much energy is released when 2.50 mg of glucose reacts with oxygen?
(A periodic table of the elements is included for your use.)

Periodic Table

1A	2A	3B	4B	5B	6B	7B	8B	8B	8B	1B	2B	3A	4A	5A	6A	7A	8A
1 H 1.008																	2 He 4.003
3 Li 6.941	4 Be 9.012											5 B 10.81	6 C 12.01	7 N 14.01	8 O 16.00	9 F 19.00	10 Ne 20.18
11 Na 23.00	12 Mg 24.31											13 Al 26.98	14 Si 28.09	15 P 30.97	16 S 32.06	17 Cl 35.45	18 Ar 39.95
19 K 39.10	20 Ca 40.08	21 Sc 44.96	22 Ti 47.90	23 V 50.94	24 Cr 52.00	25 Mn 54.94	26 Fe 55.85	27 Co 58.93	28 Ni 58.70	29 Cu 63.55	30 Zn 65.38	31 Ga 69.72	32 Ge 72.59	33 As 74.92	34 Se 78.96	35 Br 79.90	36 Kr 83.80
37 Rb 85.47	38 Sr 87.62	39 Y 88.91	40 Zr 91.22	41 Nb 92.91	42 Mo 95.94	43 Tc (98)	44 Ru 101.1	45 Rh 102.9	46 Pd 106.4	47 Ag 107.9	48 Cd 112.4	49 In 114.8	50 Sn 118.7	51 Sb 121.8	52 Te 127.6	53 I 126.9	54 Xe 131.3
55 Cs 132.9	56 Ba 137.3	57 La 138.9	72 Hf 178.5	73 Ta 180.9	74 W 183.9	75 Re 186.2	76 Os 190.2	77 Ir 192.2	78 Pt 195.1	79 Au 197.0	80 Hg 200.6	81 Tl 204.4	82 Pb 207.2	83 Bi 209.0	84 Po (209)	85 At (210)	86 Rn (222)
87 Fr (223)	88 Ra 226.0	89 Ac 227.0	104 Rf (261)	105 Ha (262)	106 Unh (263)	107 Uns (262)		109 Une (267)									

Lanthanides	58 Ce 140.1	59 Pr 140.9	60 Nd 144.2	61 Pm (145)	62 Sm 150.4	63 Eu 152.0	64 Gd 157.3	65 Tb 158.9	66 Dy 162.5	67 Ho 164.9	68 Er 167.3	69 Tm 168.9	70 Yb 173.0	71 Lu 175.0
Actinides	90 Th 232.0	91 Pa 231.0	92 U 238.0	93 Np 237.0	94 Pu (244)	95 Am (243)	96 Cm (247)	97 Bk (247)	98 Cf (251)	99 Es (252)	100 Fm (257)	101 Md (258)	102 No (259)	103 Lr (260)

Solution:

First convert 2.50 milligrams to grams.

$$2.50 \text{ mg} \times \frac{1}{1000}\frac{\text{gm}}{\text{mg}} = 0.00250 \text{ gm of } C_6H_{12}O_6$$

$$\triangle H^\circ = 0.00250 \text{ gm} \times \frac{1}{180.156}\frac{\text{mole}}{\text{gm}} \times (-2816)\frac{\text{kJ}}{\text{mole}} \approx -0.039 \text{ kJ}$$

Therefore 2.50 mg of glucose would release approximately 0.039 kJ of energy.

#3. Oscillations that are sinusoidal have an equation proportional to sine or cosine functions. Four numbers needed to describe an oscillation are the "average", "amplitude", "period", and "phase."

The *average* is the middle value (vertical coordinate) on the curve.
The *amplitude* is the difference between the maximum value and the average (or the difference between the minimum value and the average).
The *period* is the time between successive peaks.
The *phase* is the time of the first peak.

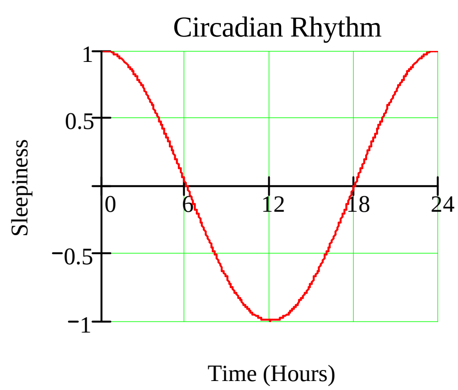
"Sleepiness" has two sinusoidal cycles, a circadian rhythm with a period of approximately 24 hours and an ultradian rhythm with a period of approximately 4 hours. Both have phase zero and average zero, but the amplitude of the circadian rhythm is 1.0 "sleepiness" units and of the ultradian is 0.4 "sleepiness" units. For the following problems assume $t = 0$ corresponds to midnight.

1. Find the circadian "sleepiness" function and sketch the graph of sleepiness over the course of a day.
2. Find the ultradian "sleepiness" function and sketch the graph of sleepiness over the course of a day.
3. Sketch the graph of the two cycles combined (circadian + ultradian).
4. To the nearest hour at what times of day is someone with the combined sleepiness function most sleepy?
5. To the nearest hour at what times of day is someone with the combined sleepiness function least sleepy?

Solution:

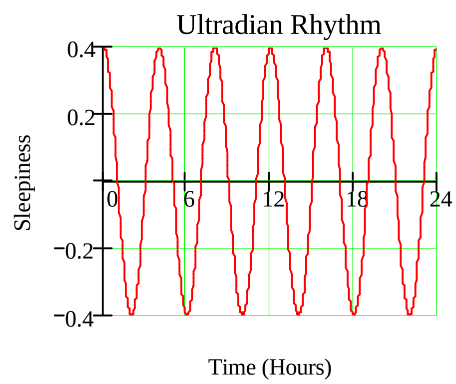
1. Let $S_C(t)$ represent the circadian rhythm.

$$S_C(t) = \cos\left(\frac{2\pi}{24}t\right) = \cos\left(\frac{\pi}{12}t\right)$$

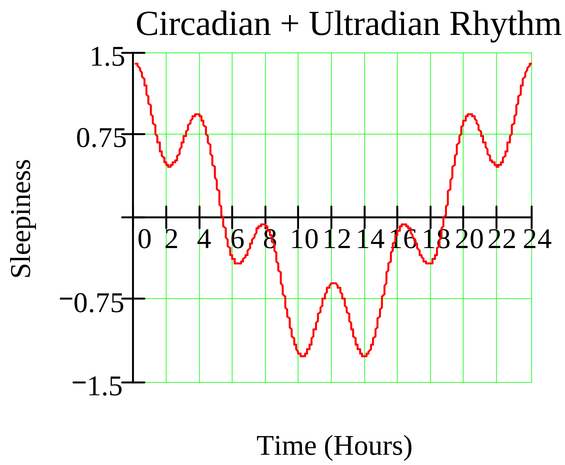


2. Let $S_U(t)$ represent the ultradian rhythm.

$$S_U(t) = 0.4\cos\left(\frac{2\pi}{4}t\right) = \cos\left(\frac{\pi}{2}t\right)$$



3. $S_C(t) + S_U(t)$

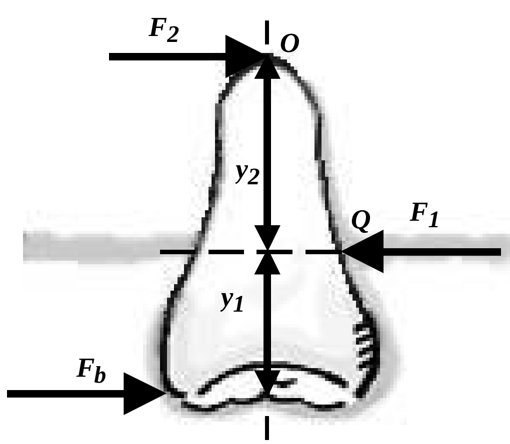


4. At midnight (12:00 A.M.) when both cycles peak together.

5. At approximately 10:00 A.M. and 2:00 P.M.

#4. Suppose that you are an orthodontist and you design a brace for a tooth. The net force exerted by the brace on the tooth is $F_b = 0.2\,\text{N}$ as shown in the figure. The force exerted on the tooth by the jawbone at point Q must not exceed $0.8\,\text{N}$, otherwise the tooth may be permanently damaged. The distances shown in the figure are $y_1 = 1.4\,\text{cm}$ and $y_2 = 1.8$ cm. The total length of the tooth is 3.2 cm.

1. What are the forces F_1 and F_2 exerted by the jawbone on the tooth at the points Q and O?
2. Is there any danger that the tooth may be damaged?



Solution:

1. The torque around pivot point O is zero.

$$\tau_O = (F_1 \times y_2) - F_b \times (y_1 + y_2) = 0$$

$$F_1 = \frac{F_b \times (y_1 + y_2)}{y_2} = \frac{0.2 \times 3.2}{1.8} \approx .356 \text{ Newtons}$$

The torque around pivot point Q is zero.

$$\tau_Q = (F_2 \times y_2) - \left(F_b \times y_1\right) = 0$$
$$F_2 = \frac{F_b \times y_1}{y_2} = \frac{0.2 \times 1.4}{1.8} \approx .156 \text{ Newtons}$$

2. Since F_1 is considerably less than 0.8 Newtons there is no danger to the tooth.

#5. Global climate change due to increasing concentrations of greenhouse gasses such as carbon dioxide could cause global temperatures to rise and this could result in widespread ecological change. In addition to broad ecosystem effects, specific impacts on human health have been predicted by such global warming. Describe three major potential consequences of global warming on human health.

Solution: (Possible answers may include the following.)

1. Increased incidence of disease:
   Many diseases such as malaria, dengue fever, filariasis, sleeping sickness, and schistosomiasis occur primarily in tropical regions. With the increase in global temperature some models predict expansion of the tropical zone which would increase the exposure to these diseases.
2. Increase in heat related deaths:
   Increased temperatures over broader regions would result in an increase in deaths from heat stroke and heat exhaustion.
3. Famine:
   There will be widespread geographic shifts in areas suitable for crop production and potentially overall reduction in land area suitable for crops. This would result in a world wide decrease in food production. Shifts in ocean currents and coastal currents would also disrupt fisheries and reduce food production from the oceans.
4. Increased storm frequency and strength:
   Storm frequency and strength is expected to increase due to temperature rises. This would be particularly significant in coastal communities but increases in tornadoes and other inland windstorms would also be significant.
5. Increase in incidence of harmful algal blooms:
   Warmer water temperatures over broader regions could result in the spread of toxic algae which would mean wider exposure for a larger portion of the human population as well as the loss of seafood resources.

#6. Cyclosporine is an immunosuppressant administered (among other applications) to prevent rejection of surgically transplanted bodily organs. The rate at which this medication is eliminated from the body (the "clearing" rate) is important in establishing the appropriate frequency of doses for patients prescribed cyclosporine. A study was undertaken to determine whether diet type (high fat, low fat) had an impact on the clearing rate of this medication. Seven healthy patients were given the drug with a low fat diet and again with a high fat diet. The clearing rates were computed for each patient on both diets and the data is summarized below. A high (or low) clearing rate is not necessarily "bad", but it is important to know whether diet type makes a difference.

	Clearing Rate	
Patient	High Fat Diet	Low Fat Diet
1	0.569	0.479
2	0.668	0.400
3	0.624	0.358
4	0.521	0.372
5	0.679	0.563
6	0.939	0.636
7	0.882	0.448

Do you think that there is sufficient evidence to indicate that diet type has an impact on the mean clearing rate for cyclosporine? Justify your answer. Quantify the degree of certainty associated with your conclusion.

Solution:

	Clearing Rate		
Patient	High Fat Diet	Low Fat Diet	Difference
1	0.569	0.479	0.090
2	0.668	0.400	0.268
3	0.624	0.358	0.266
4	0.521	0.372	0.149
5	0.679	0.563	0.119
6	0.939	0.636	0.303
7	0.882	0.448	0.434

All of the differences are positive. If diet type had no impact, the probability is 0.5 that any one difference will be positive. Thus, if diet has no impact, the probability of all seven differences being positive is $.5^7 \approx .0078$. This is a very low probability of observing the actual data. Therefore the conclusion is that diet type does have an impact on the mean clearing rate for cyclosporine.

#7. Carbon monoxide is an odorless, colorless gas that competes for sites on the red blood cells (RBCs) that carry oxygen from the blood to all other parts of the body. Carbon monoxide is a product of incomplete combustion and is produced during the burning of fossil fuels in internal combustion engines or for cooking or heating. The hemoglobin in the RBCs that carries the oxygen in the blood has a much higher binding affinity for carbon monoxide (CO) than for oxygen (O_2). When CO binds to hemoglobin, carboxyhemoglobin is formed and is unable to carry O_2. The competition between the CO and O_2 can cause health problems due to a reduction in O_2 being supplied to the body. This competition is expressed in the following Haldane equation.

$$\frac{\text{[carboxyhemoglobin]}}{\text{[hemoglobin]}} = \frac{\text{M}\times\text{[percent CO]}}{\text{[percent O}_2\text{]}}$$

In this equation M is the "binding-affinity." It has been determined to be 210. This means that CO is 210 times more likely to bind to the hemoglobin than O_2.

a. Given that the O_2 concentration in air is 21 percent, calculate the concentration of CO necessary to bind and inactivate half of the body's total hemoglobin.

Solution:

If half of the body's hemoglobin has been converted to carboxyhemoglobin then the ratio of carboxyhemoglobin to hemoglobin in the RBCs is 1.

$$\frac{\text{M}\times\text{[percent CO]}}{\text{[percent O}_2\text{]}} = 1$$

$$\text{[percent CO]} = \frac{\text{[percent O}_2\text{]}}{\text{M}} = \frac{21}{210} = 0.1$$

The CO binding-affinity is different among species. Canaries were used for many years in mines to be an indicator of atmospheres that might be toxic to the miners, since small birds have a higher metabolism and respiration rate than humans. So, generally speaking, a canary would respond to fatal conditions for humans and canaries much quicker than humans would respond to the same conditions. But the binding-affinity for CO to hemoglobin in canaries is only 105.

b. Calculate the concentration of CO necessary to inactivate half of the canary's total hemoglobin.

Solution:

$$\text{[percent CO]} = \frac{\text{[percent O}_2\text{]}}{\text{M}} = \frac{21}{105} = 0.2$$

c. Use your calculations to interpret the following chart. Discuss what happens to humans and canaries at concentrations relative to the binding of CO to hemoglobin and the metabolic rate of each organism.

Solution:

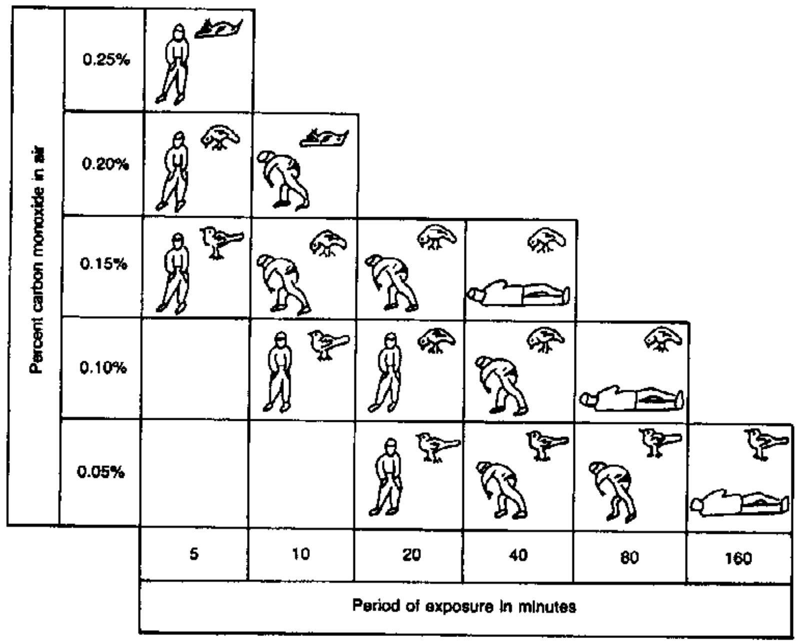


Figure 4-3. Schematic representation of how the carbon monoxide concentration in the mine air would affect the behavior of the canary or miner. Graphs are approximations of time to toxicity and lethality. (Based on Spencer, T. D. "Effects of Carbon Monoxide on Man and Canaries." *Ann. Occup. Hyg.* 5: 1961. Figure 3.)

For concentrations of CO at or above that which binds half of the hemoglobin for the canary, it will die before the human. At concentrations below that, the human is likely to die before the canary.

#8. A collagen fiber (material present in human connective tissues) has a relaxed length of 0.0347 meters. The fiber is stretched by a force that depends on the increase in length of the fiber and is given by $F(\triangle x) = k(\triangle x)^n$. The constant k is 100 Newtons per meter. The exponent n is a non-dimensional integer constant and $\triangle x$ is the increase in length of the fiber. Careful measurements show that when the fiber has tripled its length, the force has a value of 0.030 Newtons.

a. What is the value of n?
b. How much mechanical work is done by the force in stretching the fiber to its triple length?

Solution:

a.
$$F(2 \times 0.0347) = 100(0.0694)^n = 0.03\,\mathrm{N}$$
$$(0.0694)^n = \frac{0.03}{100} = 0.0003\,\mathrm{m}$$
$$n \ln(0.0694) = \ln(0.0003)$$
$$n \approx \frac{\ln(0.0003)}{\ln(0.0694)} \approx 3.04$$

Since n is an integer, the conclusion is that $n = 3$.

b. Let the distance that the fiber is stretched beyond its relaxed length be $\triangle x = s$. Then the change in the length of the fiber is from 0.0347 to 0.1041 meters. The range of s is from 0 to 0.0694 meters.

$$W = \int_0^{.0694} 100 s^3 ds = 100 \left[\frac{s^4}{4} \right]_0^{.0694} = 25 \times 0.0694^4 \approx 0.00058 \, \text{Joules}$$

#9. In an attempt to discover if high protein diets have negative effects on human beings, a group of people volunteered to eat two special foods, Nutriamite and Rabboon, for one month. Each 10-gram serving of Nutriamite contains 6 grams of protein and 4 grams of fat. Each 10-gram serving of Rabboon contains 8 grams of protein and 2 grams of fat. Each 10-gram serving of Nutriamite costs 80 cents and each 10-gram serving of Rabboon costs 75 cents. Determine the number of 10-gram servings of Nutriamite and the number of 10-gram servings of Rabboon that minimize the daily cost of the diet per person subject to the following constraints.

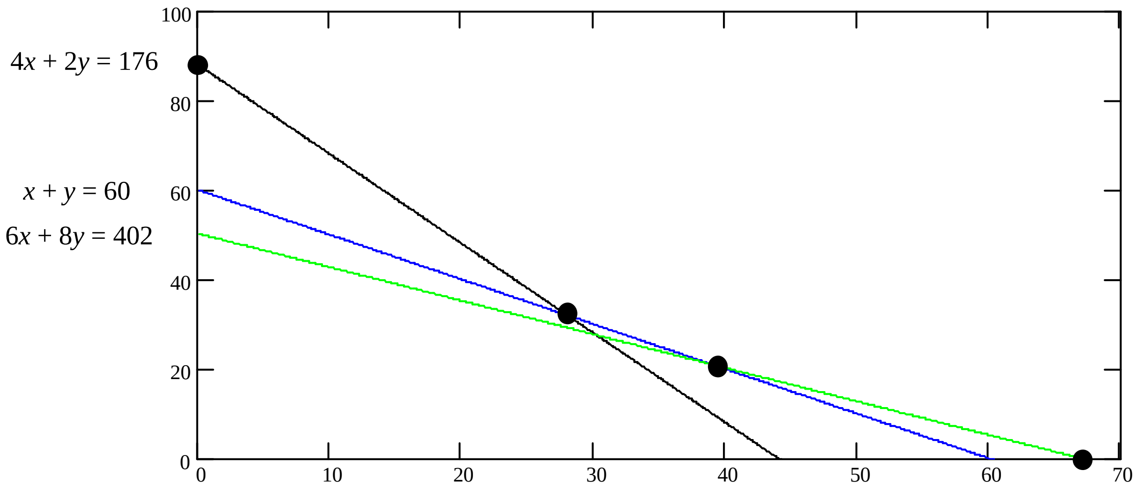
Constraint 1: Each person must eat at least sixty 10-gram servings of food each day.
Constraint 2: Each person must eat at least 402 grams of protein each day.
Constraint 3: Each person must eat at least 176 grams of fat each day.

(Note: This is a "linear programming" problem. The solution occurs at a "vertex.")

Solution:

Let x be the number of 10-gram servings of Nutriamite per day and let y be the number of 10-gram servings of Rabboon per day. The daily cost of the diet is $80x + 75y$ cents. This daily cost must be minimized subject to the following constraints.

$$x \geq 0$$
$$y \geq 0$$
$$x + y \geq 60$$
$$6x + 8y \geq 402$$
$$4x + 2y \geq 176$$



A solution occurs at one of the marked vertices.

Intersection Points:

$$\left.\begin{aligned} x + y &= \ \ 60 \\ 6x + 8y &= 402 \end{aligned}\right\} \Rightarrow (39, 21)$$

$$\left.\begin{aligned} x + y &= \ \ 60 \\ 4x + 2y &= 176 \end{aligned}\right\} \Rightarrow (28, 32)$$

Corners:

y-axis $\Rightarrow (0, 88)$
x-axis $\Rightarrow (67, 0)$

The minimum value of $80x + 75y$ subject to the given constraints occurs at one of the four points $(39, 21)$, $(28, 32)$, $(0, 88)$, and $(67, 0)$.

Vertex	$80x + 75y$
$(39, 21)$	4695
$(28, 32)$	4640
$(0, 88)$	6600
$(67, 0)$	5360

The minimum cost of $46.40 is obtained with twenty eight 10-gram servings of Nutriamite and thirty two 10-gram servings of Rabboon.